Contractionary volatility or volatile contractions?

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Abstract

There is substantial evidence that the volatility of the economy is countercyclical. This paper provides new empirical evidence on the relationship between aggregate volatility and the macroeconomy. We aim to test whether that relationship is causal. We measure volatility expectations using market-implied forecasts of future stock return volatility. According to both simple cross-correlations and a wide range of VAR specifications, shocks to realized volatility are contractionary, while shocks to expected volatility in the future have no significant effect on the economy. Furthermore, investors have historically paid large premia to hedge shocks to realized volatility, but the premia associated with shocks to volatility expectations have not been statistically different from zero. We argue that these facts are inconsistent with models in which uncertainty shocks cause contractions, but they are in line with the predictions of a simple model in which aggregate technology shocks are negatively skewed.

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1 Introduction

A wide range of measures of volatility in financial markets and the real economy are countercyclical.\footnote{Gilchrist, Sim, and Zakrajsek (2014) use the same fact as a starting point for an analysis of volatility, irreversible investment, and financial frictions. See Campbell et al. (2001) (equity volatility at the index, industry, and firm level is countercyclical); Storesletten, Telmer, and Yaron (2004) and Guvenen, Ozkan, and Song (2014) (household income risk is countercyclical); Eisfeldt and Rampini (2006) (dispersion in industry TFP growth rates is countercyclical); Alexopoulos and Cohen (2009) and Baker, Bloom, and Davis (2015) (news sources use uncertainty-related language countercyclically); among many others, some of which are discussed below.} The theoretical literature has developed numerous mechanisms through which uncertainty about the future could affect the economy, such as precautionary saving demand among households and wait-and-see behavior in firm investment.\footnote{See Basu and Bundick (2015) for a precautionary saving channel and Bloom (2009) for a wait-and-see channel.} But while theoretical models are purely about forward-looking uncertainty – increases in the breadth of the distribution of shocks that the economy faces in the future – the data that has been studied is frequently about realizations of volatility. For example, work on household- and firm-level risk almost exclusively studies how the realized distribution of shocks changes over time, rather than giving a direct measure of what people conditionally expect that distribution to look like. Similarly, while the VIX is widely studied as a measure of uncertainty about the future, it is also very highly correlated with contemporaneous volatility (Bloom (2009), in fact, uses realized volatility as a proxy for the VIX when the VIX is unavailable).

In the finance literature, it has long been understood that expected and realized volatility, while correlated, have important differences (e.g. Andersen, Bollerslev, and Diebold (2007)). A jump in stock prices, such as a crash or the response to a particularly bad macro data announcement, mechanically generates high realized volatility. On the other hand, news about future uncertainty, such as an approaching presidential election, increases expected volatility (Kelly, Pastor, and Veronesi (2016)). Furthermore, shocks to realized and expected future volatility are not nearly as strongly correlated as one might expect – in our sample, the correlation is only 65 percent.

Evidence from asset prices indicates that investors price shocks to realized and expected future volatility in the stock market differently (Egloff, Leippold, and Wu (2010); Ait-Sahalia et al. (2015); Dew-Becker et al. (2016)). Dew-Becker et al. (2016) show, in results that we extend further in this paper, that investors have historically paid large premia for protection against realized volatility, but much smaller (or even zero) premia for protection against shocks to uncertainty about the future.

The goal of this paper, then, is to understand the relationship of the business cycle with realized and expected future volatility. In addition to being interesting on its own, that question helps us understand whether the reduced-form relationship between output and volatility is causal.

We examine a sample of S&P 500 index options that has been little studied in the literature, but that allows us to measure volatility expectations since 1983 at horizons of up to 6 months. We thus have a sample spanning 32 years, three recessions, and a wide range of financial and real conditions in the economy. While options have the drawback that their prices also depend on risk premia, we take a number of steps to show both theoretically and empirically that our results are robust to the
presence of risk premia. We measure realized volatility as the sum of daily squared returns on the market each month. This concept differs from that used by some other recent papers, but has a number of advantages, including directly measuring the concept of interest (the realized dispersion in aggregate shocks) and being linked in a well-defined way to our volatility forecasting variable. While there are many different potential measures of volatility and uncertainty in the economy, if one wants to cleanly distinguish between expectations and realizations, equity indexes may be the best source available.

Using both cross-correlations and VARs, we find that increases in realized volatility are associated with declines in output, consumption, investment, and employment, consistent with findings in Bloom (2009) and Basu and Bundick (2015). More surprisingly, though, increases in six-month expected volatility have no significant effect on the real economy across a range of specifications – some specifications imply expected volatility shocks are mildly contractionary but insignificant, others that they are actually expansionary, sometimes even significantly so.

In addition, a forecast error variance decomposition shows that shocks to expected volatility account for less than 1 percent of the variance of employment and industrial production at all horizons; the 97.5 percentile of this estimate is less than 5 percent for horizons up to a year. Expected volatility shocks – as captured by our measure – do not seem to be an important source of macroeconomic fluctuations.

The difference in the effects of shocks to realized and expected volatility is itself statistically significant. In other words, increases in uncertainty about future stock market performance do not appear to reduce output, even though realized volatility tends to be high in bad times. The results are consistent in finding significant effects for realized volatility and insignificant effects for volatility expectations across estimation methods and they are robust to alternative identification schemes, changing the data sample and state variables, choices about detrending, and estimating the model at the quarterly or monthly frequency.

The simplest theoretical explanation for our results is that fluctuations in economic activity are negatively skewed. That could either be because the fundamental shocks are skewed, or because symmetrical shocks are transmitted to the economy asymmetrically (e.g. because constraints, such as financial frictions, bind more tightly in bad times; Kocherlakota (2000)). Skewness literally says that the squared value of a variable is correlated with the variable itself, which is essentially what we find when realized volatility is associated with contractions.

There are two important pieces of evidence in favor of the skewness hypothesis. First, changes in a wide variety of measures of real activity are negatively skewed, as are stock returns. Second, the asset prices we examine imply that investors have paid large premia for insurance against high realized volatility and extreme negative stock returns (known as the variance risk premium and the option skew or put premium, respectively) in the last 30 years, whereas the premium paid for protection against increases in expected volatility has historically been near zero or even positive.

Finally, we also build an equilibrium model that features both skewed shocks and variation in uncertainty. It is a model with a time-varying probability of medium-size downward jumps in
productivity.\(^3\) We show that VARs estimated in simulations of the model deliver estimates that are highly similar to what we observe empirically – the small disasters themselves induce recessions and high realized volatility, while variation in the conditional variance of shocks has almost no effect on output. The model also matches the risk premia that we observe for shocks to expected and realized volatility.

To summarize, then, we provide evidence from VARs, the term structure of variance risk premia, the skewness of real activity, and a structural model of the economy that suggests that output and realized volatility in the stock market are jointly caused by negatively skewed fundamentals.

Two important caveats apply to our results. First, our analysis is of the effects of fluctuations in aggregate stock market uncertainty. It simply shows that the VIX, after controlling for current conditions (realized volatility) does not have predictive power for the future path of the economy. We do not measure variation in cross-sectional uncertainty. There are obviously many dimensions along which uncertainty can vary, and we try to understand just one here.

The second important caveat is that there are other measures of aggregate uncertainty that may still cause contractions even after controlling for realized volatility. The key contribution of this paper is to show that when a single concept of volatility can be split into components coming from realizations and expectations, it is the realization component that appears to drive the results. That does not imply, though, that no other measures of uncertainty (which do not distinguish between expectations and realizations) can affect the economy.\(^4\)

Our work is related to a large empirical literature that studies the relationship between aggregate volatility and the macroeconomy.\(^5\) A number of papers use VARs, often with recursive identification, to measure the effects of volatility shocks on the economy.\(^6\) Ludvigson, Ma, and Ng (2015), like us, distinguish between different types of uncertainty. They show that variation in uncertainty about macro variables is largely an endogenous response to business cycles, whereas shocks to financial uncertainty cause recessions. The key distinction between our work and theirs is that we focus on the distinction between uncertainty expectations and realizations, while they distinguish between uncertainty in different sectors of the economy.

This paper is also related to a growing literature exploring whether higher uncertainty causes recessions. Most of this work as focused on the relationship between idiosyncratic uncertainty and the macroeconomy. Baker and Bloom (2013) use cross-country evidence to argue that there is causal and negative relationship between uncertainty and growth. Recently, there is a growing literature arguing that causality runs the opposite direction: measured volatility could be an en-

\(^3\) It is conceptually similar to Gourio (2012), but with smaller and more frequent “disasters”, consistent with the evidence in Backus, Chernov, and Martin (2011).

\(^4\) For models with different types of uncertainty see for example Segal et al. (2015).


\(^6\) See Bloom (2009) and Basu and Bundick (2015), who study the VIX; and Baker, Bloom, and Davis (2015) and Alexopoulos and Cohen (2009), who study news-based measures of uncertainty. Jurado, Ludvigson, and Ng (2015) and Ludvigson, Ma, and Ng (2015) measure uncertainty based on squared forecast errors for a large panel of macroeconomic time series (using a two-sided filter to extract a latent volatility factor).
dogenous response to negative first moment shocks (Decker, D’erasmo and Boedo (2016), Berger and Vavra (2013), and Ilut, Kehrig and Schneider (2015)).

Last, since our empirical analysis uses option prices to infer investor expectations of future volatility in the economy, it draws from and builds on a large literature in finance examining the pricing and dynamics of volatility.\footnote{See, among many others, Adrian and Rosenberg (2008), Bollerslev et al. (2009), Heston (1993), Ang et al. (2006), Carr and Wu (2009), Bakshi and Kapadia (2003), Egloff, Leippold, and Wu (2010), and Ait-Sahalia, Karaman, and Mancini (2013) (see Dew-Becker et al. (2016) for a review).}

The remainder of the paper is organized as follows. Section 2 presents the evidence on risk premia associated with contracts that hedge variance shocks. Section 3 lays out a simple reduced-form model of financial markets and the real economy that helps elucidate the distinction between realized and expected volatility. Sections 4 and 5 describe our data and examine its basic characteristics. Section 6 describes our main analysis of the relationship between realized volatility, expected volatility, and the real economy. Finally, section 7 proposes a structural equilibrium model that matches the stylized facts on the skewness of shocks, the pricing of volatility claims, and the response of the economy to volatility shocks. Section 8 concludes.

## 2 Buying insurance against volatility shocks

Derivative contracts allow investors to construct assets whose payoffs are nonlinear functions of stock returns. The prices of those derivatives can then be used to measure the premia that investors are willing to pay for protection from (or exposure to) different types of shocks to the economy. Of particular interest to us are the premia that investors are willing to pay for protection against shocks to stock market volatility.

A one-month variance swap is an asset whose final payoff depends on the sum of daily squared log returns of the underlying index (the S&P 500, in our case) over the next month. That asset gives the buyer protection against a surprise in equity return volatility over the next month. The market price obviously reflects expected volatility (and a risk premium), and any surprise in the return reflects a surprise in volatility. When volatility is unexpectedly high, the buyer earns a relatively high return. If investors are averse to periods of high realized volatility, then, we would expect to see negative average returns on one-month variance swaps, reflecting the cost of buying that insurance. A simple way to see that is to note that under power utility, for any asset return $x$, the Sharpe ratio is

$$
\frac{ER[x]}{SD[x]} = RRA \times std(\Delta c) \times corr(x, \Delta c)
$$

where $RRA$ is the coefficient of relative risk aversion. Under power utility, assets with high Sharpe ratios have high correlations with consumption growth.

The first point on the left in the left-hand panel of Figure 1 plots average Sharpe ratios (mean excess returns divided by the standard deviation of returns, both annualized) on 1-month S&P
500 variance swaps between 1996 and 2014. The average Sharpe ratio is -1.4, approximately three times larger (with the opposite sign) than the Sharpe ratio on the aggregate equity market. In other words, investors have been willing to pay extremely large premia for protection against periods of high realized volatility, suggesting that they view those times as particularly bad (or as having very high marginal utility).

Now consider a $j$-month variance forward, whose payoff, instead of being the sum of squared returns over the next month $(t + 1)$, is the sum of squared returns in month $t + j$ (so then the one-month variance swap above can also be called a 1-month variance forward). If an investor buys a $j$-month variance forward and holds it for a single month, selling it in month $t + 1$, then the variance forward protects them over that period against news about volatility in month $t + j$. That is, if we get news that volatility will be higher in the future, it will affect the holding period return on that $j$-period forward. Figure 1 (left panel) also plots one-month holding period Sharpe ratios for variance forwards with maturities from 2 to 12 months. We see that for all maturities higher than 2 months, the Sharpe ratios are near zero, and in fact the sample point estimates are positive. The Sharpe ratios are also all statistically significantly closer to zero than the Sharpe ratio on the one-month variance swap.

Figure 1 therefore shows that there is something special about the surprise in realized volatility compared to news about volatility going forward. Investors are willing to pay large premia for protection against surprises in realized volatility, but news about future uncertainty has a much smaller – or even zero – premium.

Using the options data we will describe below, it is possible to extend those results further, back to 1983. The right-hand panel of figure 1 reports the average shape of the term structure of variance forward prices constructed from data on S&P 500 options (we study the term structure with this data because it is estimated more accurately than returns and it is what will enter our empirical analysis below). The term structure reported here is directly informative about risk premia. The average return on an $n$-month variance claim is:

$$E \left[ \frac{F_{n-1,t} - F_{n,t-1}}{F_{n,t-1}} \right] \approx \frac{E \left[ F_{n-1} \right] - E \left[ F_n \right]}{E \left[ F_{n} \right]}$$

where $F_{n,t}$ is the price on date $t$ to a claim to realized stock market volatility in month $t + n$. The slope of the average term structure thus indicates the average risk premium on news about volatility $n$ months forward. If the term structure is upward sloping, then the prices of the variance claims fall on average as their maturities approach, indicating that they have negative average returns. If it slopes down, then average returns are positive.

The right-hand panel of Figure 1 plots the average term structure of variance forward prices for

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8The data is described in Dew-Becker et al. (2016); it is obtained from a large asset manager and Markit, but may be closely approximated by portfolios of options, for which prices are widely available (e.g. from Optionmetrics)
the period 1983–2013. The term structure is strongly upward sloping for the first two months, again indicating that investors have paid large premia for assets that are exposed to realized variance and expected variance one month in the future. But the curve quickly flattens, indicating that the risk premia for exposure to fluctuations in expected variance further in the future have been much smaller.

The asset return data says that investors appear to have been highly averse to news about high realized volatility, while shocks to expected volatility have not seemed related to marginal utility. The confidence intervals that we obtain are sufficiently wide that we cannot claim that shocks to expected future volatility do not earn an economically meaningfully negative risk premium. What we can say, though, is that investors seem to have cared over our sample much more about surprises in realized volatility than in expected volatility.

Motivated by figure 1, this paper asks whether the relationships of realized volatility and expected future volatility with output are also different. The obvious implication of figure 1 is that investors think that realized volatility is associated with downturns, while news about expected future volatility (i.e. an uncertainty shock) is not.

3 Empirical model

We begin with a stylized and largely reduced-form model of output and stock returns to help elucidate the VARs that we estimate and their source of identification. A central feature of the model is that it has distinct roles for expected and realized volatility, consistent with the results on risk premia in the previous section.

We think of the data as being driven by a continuous-time process that is sampled discretely, e.g. on a monthly or quarterly basis. Suppose monthly samples of stock returns follow the process

\[ R_t = \sigma_{r,t-1} \varepsilon_{r,t} + \varepsilon_{J,t} \]  

(3)

where \( \varepsilon_{r,t} \) is a standard normal innovation (representing the accumulation of an orthogonal increment process, for example). \( \sigma_{r,t-1} \), known at the end of period \( t - 1 \), determines the volatility of returns.

\( \varepsilon_{J,t} \) is a jump shock with \( \varepsilon_{J,t} = J \times (N_t - \lambda \sigma_{r,t-1}^2) \) and \( N_t \) drawn from a Poisson distribution with intensity (and variance) \( \lambda \sigma_{r,t-1}^2 \). \( \varepsilon_{J,t} \) is independent of the other shocks and has zero mean by construction.\(^9\)

Denote the vector of state variables in the economy \( X_t \). We assume that output, \( Y_t \), loads on the same shocks as returns, and may also have an independent component, \( \varepsilon_{Y,t} \)

\[ Y_t = k_{Y,J} \varepsilon_{J,t} + k_{Y,r} \varepsilon_{r,t} + \varepsilon_{Y,t} + k_{Y,\sigma} \sigma_{r,t} + \Phi_Y(L) X_{t-1} \]

(4)

\(^9\)In continuous time, the return process during period \( t \) is \( R_t^{cont} = \sigma_{t-1} dZ_t + J (dN_t - \lambda \sigma_{t-1}^2 dt) \), where \( dZ_t \) is an orthogonal increment process, \( dN_t \) is a Poisson process with intensity \( \lambda \sigma_{t-1}^2 \), and \( t \) indexes time within period \( t \). \( R_t \) is then the integral of \( R_t^{cont} \) over the period \( t \).
where $\Phi_Y(L)$ is a polynomial in the lag operator $L$. $k_{Y,J}$ is a coefficient determining the impact of the jump shock on output. If $k_{Y,J} > 0$ and $J < 0$, we can think of this as a model where there are infrequent negative shocks that induce left skewness in output and returns. So it can be interpreted as a setting with sharp contractions. Most importantly, volatility expectations, $\sigma_{\tau,t}$, can directly affect output if $k_{Y,\sigma} \neq 0$. $k_{Y,\sigma}$ is interpreted as the structural effect of volatility expectations (uncertainty) on output, and it is thus our key parameter of interest.

The realized volatility of stock returns, calculated as the sum of squared returns sampled at high (e.g. daily) frequency, is,

$$RV_t = \sigma_{r,t}^2 + J (\varepsilon_{J,t} - J\lambda \sigma_{r,t}^2)$$

(5)

We assume that $\sigma_{r,t}^2$ follows an ARCH-type process (Engle (1982); Bollerslev (1986)) in which realized volatility in period $t$ may affect volatility in the future,

$$\sigma_{r,t}^2 = \bar{\sigma}^2 + k_{\sigma,rv} RV_t + \varepsilon_{\sigma,t}$$

(6)

$\varepsilon_{\sigma,t}$ is a mean-zero innovation that represents news about future volatility that is independent of current realized volatility. $k_{\sigma,rv}$ determines the extent to which realized volatility in period $t$ feeds back into volatility in future periods. There is a very large finance literature on the persistence of volatility and how it can feed back on itself.\(^\text{11}\) The idea here is that expected volatility during period $t$ depends on the realization of volatility in period $t - 1$ – a crash in January is expected to lead to above average volatility in February also.\(^\text{12}\)

As discussed in the introduction, the basic starting point for the uncertainty literature is the observed negative covariance between output and uncertainty. In the present model we have

$$\text{cov}_{t-1} (Y_t, \sigma_{r,t}^2) = k_{Y,\sigma} \left( (k_{\sigma,rv} J)^2 \text{var} (\varepsilon_{J,t}) + \text{var} (\varepsilon_{\sigma,t}) \right) + k_{Y,J} k_{\sigma,rv} J \text{var} (\varepsilon_{J,t})$$

(9)

The first term represents the covariance due to the structural effect of uncertainty on output, and it is only nonzero if $k_{Y,\sigma}$ is non-zero. The second term, though, can be non-zero even if there is no structural effect of uncertainty on output. It appears because output and uncertainty are both affected by the jump shock $\varepsilon_{J,t}$. So $\text{cov}_{t-1} (Y_t, \sigma_{r,t}^2)$ does not identify the impact of uncertainty

\(^{10}\)Technically, our definition of $RV_t$ holds when the sampling frequency for realized volatility becomes infinitely high. In terms of the continuous-time specification, $RV_t$ is the integral of $(R_{cont}^2)^2$ over the period $t$.

\(^{11}\)See Engle (1982), Bollerslev (1986), and the literature following them.

\(^{12}\)The model of stock returns has a stark division between the shock to realized volatility, $\varepsilon_{J}$, and the independent shock to volatility expectations, $\varepsilon_{\sigma}$. That distinction is purely for simplicity. The identification scheme that we describe below works identically if, alternatively, there are two shocks that both affect realized volatility and expected future volatility, but in different proportions. Specifically, $RV_t$ and $\sigma_{r,t}^2$ could follow the processes

$$RV_t = \sigma_{r,t-1}^2 + \varepsilon_{r,t}^{(1)} + \varepsilon_{r,t}^{(2)}$$

$$\sigma_{r,t}^2 = \bar{\sigma}^2 + k_{\sigma,rv} \sigma_{r,t-1}^2 + k_{\sigma,d} \varepsilon_{r,t}^{(1)} + k_{\sigma,2} \varepsilon_{r,t}^{(2)}$$

(7)

(8)

The necessary assumption in that case is that $k_{\sigma,1} \neq k_{\sigma,2}$ – i.e. that the two shocks have different pass-through to future volatility. For the sake of parsimony, though, our main analysis follows the specification in equations (5–6).
on output. The reason can be interpreted as omitted variable bias – we need to control for $\varepsilon_{J,t}$ somehow because both $Y_t$ and $\sigma^2_{r,t}$ are correlated with it.

A way to control for $\varepsilon_{J,t}$, and thus identify $k_{Y,\sigma}$, is to examine the covariance of output and uncertainty conditional on current realized volatility. Specifically, conditioning on $RV_t$ yields

$$\text{cov}_{t-1} \left( Y_t, \sigma^2_{r,t} \mid RV_t \right) = k_{Y,\sigma} \text{var} \left( \varepsilon_{\sigma,t} \right)$$ (10)

We now have the result that the sign of the conditional covariance between output and uncertainty reveals the structural relationship. $\text{cov}_{t-1} \left( Y_t, \sigma^2_{r,t} \mid RV_t \right)$ can also be thought of as proportional to the coefficient on $\sigma^2_{r,t}$ in a regression of $Y_t$ on $\sigma^2_{r,t}$ and $RV_t$.\(^{13}\)

In the data, we will not be able to directly measure the conditional expectation of realized variance, $RV_t$. Instead, we only measure an option-implied expectation of volatility, which we denote $EV_t$. $EV_t$, since it is an asset price, can depend on risk premia. We account for that possibility by assuming that it is a linear function of the true statistical expectation of volatility, realized volatility, and lagged information

$$EV_t = k_{EV,0} + k_{EV,\sigma}\sigma^2_{r,t} + k_{EV,RV}RV_t + \Phi_\sigma \left( L \right) X_{t-1}$$ (12)

(12) means that the risk premium in option-implied volatility can be correlated with $EV_t$, $\sigma^2_{r,t}$, $RV_t$, and the history of the state variables. We confirm in the empirical analysis that variables other than option-implied volatility and current realized volatility do not help forecast future volatility, consistent with (12).

A simple way to measure the conditional covariance in (10) is with a VAR. Setting $X_t \equiv \left[ RV_t, EV_t, Y_t \right]'$, the above assumptions imply that $X_t$ follows

$$X_t = C + \Phi \left( L \right) X_{t-1} + \begin{bmatrix} 1 & 0 & 0 \\ k_{EV,\sigma}k_{\sigma,rv} + k_{EV,RV} & 1 & 0 \\ k_{Y,J} + k_{Y,\sigma}k_{\sigma,rv} & k_{Y,\sigma}k_{EV,\sigma}^{-1} & 1 \end{bmatrix} \begin{bmatrix} J\varepsilon_{J,t} \\ k_{EV,\sigma}\varepsilon_{\sigma,t} \\ \tilde{\varepsilon}_{Y,t} \end{bmatrix}$$ (13)

where $C$ is a vector of constants, $\Phi \left( L \right)$ is a matrix lag polynomial, and $\tilde{\varepsilon}_{Y,t}$ is the residual from a projection of the innovation in $Y_t$ onto $\varepsilon_{J,t}$ and $\varepsilon_{\sigma,t}$. The impact matrix multiplying the vector of shocks can be measured with a Cholesky factorization of the covariance matrix of the reduced-form innovations in the VAR. The second identified “shock” represents simply the innovation in the implied volatility, $EV_t$, conditional on $RV_t$ (which is proportional to $\varepsilon_{\sigma,t}$) – exactly what we desire.

\(^{13}\) A restriction in the model above is that there is no feedback from output to volatility. If we allow such a channel, the conditional covariance between output and uncertainty becomes

$$\text{cov}_{t-1} \left( Y_t, \sigma^2_{r,t} \mid RV_t \right) = k_{Y,\sigma} \text{var} \left( \varepsilon_{\sigma,t} \right) + k_{\sigma,Y} \text{var} \left( \varepsilon_{Y,t} \right)$$ (11)

where $k_{\sigma,Y}$ is the response of uncertainty to $\varepsilon_{Y,t}$. If we assume that $k_{\sigma,Y} < 0$, i.e. an increase in output reduces uncertainty (as in, e.g., van Nieuwerburgh and Veldkamp (2006) or Ludvigson, Ma, and Ng (2016)), then if $\text{cov}_{t-1} \left( Y_t, \sigma^2_{r,t} \mid RV_t \right)$ is estimated to be non-negative, then so is $k_{Y,\sigma}$. In other words, if there is potentially feedback from output to volatility, then we must impose the sign restriction that $k_{\sigma,Y} > 0$. 

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to identify in equation (10). So the loadings on that second shock identify conditional covariances with volatility news, $\varepsilon_{t,\sigma}$. For our purposes, what is important is that the loading of output on that conditional innovation is proportional to $k_{Y,\sigma}$, exactly as in equation (10).\footnote{It is important to note that Bloom (2009) controls for market returns in the VAR first – replacing $RV$ in our setting (though Basu and Bundick (2015) do not). Since the market return depends on shocks other than $\varepsilon_{J,t}$, controlling for the return does not completely control for $\varepsilon_J$ in the way that $RV$ does. That means that the Cholesky-identified “shock” to $EV_t$ in that case still contains information about $\varepsilon_{J,t}$, potentially leading to a spurious correlation with output.}

We will therefore estimate a VAR using a Cholesky factorization to measure conditional covariances. In our main analysis, we include more than a single measure of output (the main VAR includes realized volatility, an option-implied volatility, the Fed Funds rate, industrial production, and employment). Our results are partially identified in the sense that we order the shock to realized variance before the shock to expected volatility but leave the ordering of all the other variables unconstrained (see also Christiano, Eichenbaum, and Evans (2005)). We do not claim that true structural shocks satisfy any kind of timing restriction. In the empirical section we discuss the implications of alternative orderings for the identification.

The simple model in this section illustrates our basic goal in estimating the covariance of output with uncertainty ($EV_t$) conditional on realized volatility and helps elucidate the key operating mechanisms. First, since high realized volatility is associated with high expected volatility, conditioning on $RV_t$ (i.e. ordering it in a VAR before $EV_t$) is critical in testing the hypothesis that expected volatility shocks can affect the economy. True uncertainty shocks – changes in volatility expectations that are orthogonal to shocks to other variables – are identified in the VAR as the part of $EV_t$ that cannot be explained by its own lag or by $RV_t$. Past work has typically included only $RV_t$ or $EV_t$ (or analogous variables) individually, but not the two together, so it could not distinguish whether volatility causes declines in output or is simply correlated with negative shocks to the economy.

The second important mechanism in the model is that realized volatility can be correlated with output because it isolates the large shock, $\varepsilon_{J,t}$, that is common to output and financial markets. In the model, there are small shocks, $\varepsilon_{Y,t}$ and $\varepsilon_{r,t}$, that affect the real and financial sides of the economy, respectively, and obscure the relationship between output and stock returns. But when big shocks hit the economy, they tend to affect both output and stock returns. $RV_t$, by looking at squared returns, isolates the large shocks in stock returns, essentially stripping out the diffusive noise (especially after controlling for lagged volatility expectations).

4 Data

The concept of volatility studied in this paper is aggregate equity return volatility. We focus on stock market volatility for a number of reasons.

The feature of the data that we desire to measure is the variance of the flow of aggregate shocks that hit the economy. We thus do not aim to measure cross-sectional dispersion in shocks or even
forecast uncertainty. Jurado, Ludvigson, and Ng (2015), for example, construct a monthly measure of forecast uncertainty for a wide range of macroeconomic variables. Our goal, on the other hand, is to measure the variance of the common shocks to measures of activity, rather than the total dispersion of each measure.

Equity prices summarize information about the future path of the economy, so volatility in the economy should be expected to be related to volatility in the stock market. One would expect that almost any factor that affects risk in the economy would affect the riskiness of firms, since the revenue and profitability of firms ultimately depend on all the features of the economy.\footnote{For example, in standard investment theories, stock prices are closely related to the discounted present value of the marginal product of capital (in q theory, that link is exact).}

4.1 Model-free implied volatility ($EV_n$)

Equity return volatility is measured in this paper as the volatility of returns on the S&P 500 index. In particular, realized volatility in each month is measured as the sum of squared daily returns. We denote realized volatility in month $t$ as $RV_t$. We obtain data on daily stock returns from the CRSP database, which has coverage since 1926.

We construct measures of expected future volatility, $EV_t$, using option prices. The prices of S&P 500 options are obtained from the Chicago Mercantile Exchange (CME), with traded maturities from 1 to at least 6 months since 1983.

The most widely used measure of expected volatility is the VIX. The VIX is an index, based on option prices, of implied volatility over the next 30 days. There are two main issues with using the VIX as a measure of expected future volatility in our empirical analysis. First, since realized volatility is persistent, one-month expectations are significantly correlated with realizations (e.g. due to the ARCH effects in the model above). But at longer horizons, we observe larger differences between realized and expected volatility, allowing for their effects to be separately identified (i.e. allowing us to measure $\varepsilon_{\sigma,t}$).

Moreover, the 30-day maturity of the VIX makes it less than ideal to capture the kind of uncertainty about the future that might matter to firms and consumers, whose horizons are typically much longer.

These problems with the VIX are solved by examining implied volatilities at longer maturities. From a theoretical point of view, the expectation of future volatility 6 or 12 months ahead lines up more closely with firms' decision frequency in macroeconomic models.

In this section, we describe how we construct a version of the VIX (the price of a claim to future realized volatility), extending it beyond the usual one-month horizon. We will refer to the extension of the VIX to arbitrary maturities as model-free implied volatility, or $EV_n$ (expected volatility), where $n$ refers to the horizon in months. The 30-day VIX is then $EV_1$.

The VIX and $EV_n$ are claims to future realized volatility. The fundamental time period that we analyze in this paper is a single month, which is indexed by $t$. Within any month, the realized
volatility of equity returns is calculated as

\[ RV_t \equiv \left( \sum_{\text{days } i \in t} r_i^2 \right)^{1/2} \] (14)

where \( r_i \) is the log return on the S&P 500 on day \( i \) in month \( t \) (note that from here on \( RV_t \) is in standard deviation units, rather than the variance units used above, for consistency with the past literature).

There is a large literature in finance that examines model-free implied volatilities for various assets. Under very general conditions (Jiang and Tian (2005) and Carr and Wu (2009)), the price of a claim to realized volatility between dates \( t + 1 \) and \( t + n \) can be written as a function of time-\( t \) option prices:

\[ EV_{n,t} = \left( 2 \int_0^\infty \frac{O_t(n,K)}{B_t(n)K^2} dK \right)^{1/2} \approx \left( E_t \left[ \sum_{m=1}^n \frac{M_{t,t+m} RV_{t+m}^2}{E_t M_{t,t+m}} \right] \right)^{1/2} \] (15)

where \( M_{t,t+m} \) represents the stochastic discount factor between dates \( t \) and \( t + m \).

Model-free implied volatility \( EV_n \) is calculated as an integral over option prices, where \( K \) denotes strikes, \( O_t(n,K) \) is the price of an out-of-the-money option with strike \( K \) and maturity \( n \), and \( B_t(n) \) is the price at time \( t \) of a bond paying one dollar at time \( t + n \). \( EV_{n,t} \) is approximately equal to total expected volatility over the next \( n \) months (to be most precise, model-free implied volatility is the square root of expected variance; \( RV^2 \) is used since variances are additive over time). The approximate equality is related to the discretization in the calculation of realized variance.

The model-free implied volatility makes few assumptions about the dynamics of stock returns, which is why \( EV_{1,t} \) is used by the CBOE as its definition of the VIX. Crucially, unlike the Black–Scholes (1973) implied volatility, it does not require that volatilities be constant over time.\(^{16}\)

Expectations involving the term \( \frac{M_{t,t+n}}{E_t M_{t,t+n}} \) are often referred to as risk-neutral, or risk-adjusted expectations,

\[ E_t^Q [X_{t+1}] \equiv E_t \left[ \frac{M_{t,t+1}}{E_t M_{t,t+1}} X_{t+1} \right] \] (16)

The model-free implied volatility (and hence the VIX) is a risk-neutral or market-implied expectation of future realized volatility. A risk-neutral expectation depends on both the physical expectation of future volatility and also any risk adjustment due to covariation with the pricing kernel (state prices). As discussed in the theoretical analysis above, our identification scheme does not require any assumption that risk premia are constant.

We construct the model-free implied volatility, \( EV_n \), for the S&P 500 at maturities between 1 and 6 months. We calculate it using data on option prices from the Chicago Mercantile Exchange

\(^{16}\)Martin (2015) provides an alternative implied variance that involves even fewer restrictions than that we use here. We have replicated our analysis with his measure and find nearly identical results. We focus on the VIX-type measure simply because it has been widely studied in the past.
(CME), which allows us to construct a time series dating back to 1983. Throughout the paper and appendix, we examine a range of robustness tests, including constructing volatility expectations using alternative sources of data on option prices and using variance swaps, whose payoffs are directly linked to realized volatility (but for which the sample is only half as long). Computing the model-free implied volatility with real-world data requires several steps; the appendix provides an extensive description of our calculation methods and analyzes the accuracy of the data.

Finally, in the remainder of the paper we focus on the logs of realized volatility \((RV)\) and 6-month expected volatility \((EV_6)\). Given the high skewness of realized variance, the log transformation makes the results less dependent on the occasional volatility spikes. We nonetheless show that our results are also robust to running the analysis in levels. We refer to variables in logs everywhere with lower-case letters, e.g. \(rv_t = \log RV_t\).

### 4.2 The time series of realized volatility and its expectations

Figure 2 plots the history of (annualized) realized volatility along with 6-month market expectations \((EV_6, \text{ also annualized})\). Both realized volatility and volatility expectations vary considerably over the sample. The two most notable jumps in volatility are the financial crisis and the 1987 market crash, which both involved realized volatility above 60 annualized percentage points and rises of the 6-month expectations to 40 percent. At lower frequencies, the periods 1997–2003 and 2008–2012 are associated with persistently high volatility expectations, while expectations are lower in other periods, especially the early 1980’s, early 1990’s, and mid-2000’s. There are also distinct spikes in expected volatility in the summers of 2010 and 2011, likely due to concerns about the stability of the Euro and the willingness of the United States to continue to pay its debts.

Panel A of Table 1 reports descriptive statistics for the series in figure 2. The mean of volatility expectations is substantially higher than that of realized volatility, which is due to the risk-adjustment mentioned above. Specifically, there is a negative risk premium on volatility (Coval and Shumway 2001), which causes the prices of financial claims on volatility to be upward biased estimates of future volatility. As we would expect, the standard deviation of expectations is smaller than that of realized volatility.

Panel B of Table 1 reports raw correlations of the logs of realized and 6-month expected volatility with measures of real economic activity – capacity utilization, the unemployment rate, and returns on the S&P 500 (correlations are similar in levels). Volatility is correlated with all three macroeconomic variables, most strongly with capacity utilization; what the rest of this paper will explore is whether this association is due to simple correlation with realized volatility, or whether expected future volatility is causally linked to macroeconomic activity.
5 The dynamics of volatility expectations

We now examine the dynamics of realized volatility and its market expectations more formally. We first estimate simple forecasting regressions to measure how well realized volatility is predicted by lagged market expectations. It is obviously critical that we show that model-free implied volatility actually forecasts volatility, otherwise we cannot claim to identify volatility news, $\varepsilon_{\sigma,t}$.

5.1 Forecasting regressions for realized volatility

The top panel of Figure 3 plots the coefficient $\beta_h$ in a special case of the predictive relationship between $ev$ and $rv$ from the model in equation (12)

$$rv_t = \alpha + \beta_h ev_{6,t-h} + \varepsilon_t$$

for different lags $h$ (on the x-axis). The figure shows that market-implied expected volatility indeed has highly significant forecasting power for future realized volatility, with a statistically and economically significant coefficient even 12 months ahead. The coefficients are well below 1, though, which implies that $ev_{6}$ is not a pure statistical expectation of future realized volatility, which could be due to both measurement error and time-varying risk premia.

Since $rv$ is a persistent process, and since there are well known ARCH effects in the data, a critical question is whether $ev_{6}$ contains any information about future $rv$ after controlling for $rv$ itself. We therefore estimate the multivariate specification,

$$rv_t = \alpha + \beta_h ev_{6,t-h} + \gamma_h rv_{t-h} + \varepsilon_t$$

$\beta_h$ now measures whether market expectations have any predictive content for future realized volatility after controlling for lagged volatility. In order for $\beta_h$ to be non-zero, there must be shocks to volatility expectations that are independent of ARCH effects, i.e. $std(\varepsilon_{\sigma,t}) \neq 0$.

The second row of Figure 3 shows the two sets of coefficients, $\beta_h$ and $\gamma_h$, for different lags $h$. The left panel shows the coefficient on lagged $rv$, $\gamma_h$. As expected, lagged $rv$ forecasts future $rv$, with a coefficient declining with the horizon. More interestingly, though, the right panel shows that $ev$ does in fact have significant predictive power for future volatility at all horizons, even after controlling for lagged $rv$. Moreover, the coefficients and t-statistics for $rv$ and $ev_{6}$ are similar at all horizons, indicating that they have similar marginal $R^2$s. That is, the two variables have roughly the same amount of marginal predictive power in all the regressions. So Figure 3 shows that there are in fact volatility news shocks, $\varepsilon_{\sigma,t}$, they are statistically measurable, and they can be identified from option prices.

Table 2 expands the analysis by looking at realized volatility over 6-month horizons and adding a set of additional predictors. In particular, columns 1 and 2 repeat the analysis of Figure 3, demonstrating the predictive power of $ev_{6}$ for future 6-month realized volatility. Column 3 adds
three macroeconomic variables (industrial production, employment, and the federal funds rate) that we will use in our VAR analysis below. The table shows that the forecasting power of $ev_6$ for future realized volatility survives adding many variables. The macroeconomic indicators do not have any significant forecasting power for future volatility – their t-statistics are all smaller than 0.5, and the Bayesian information criterion also implies that they should be excluded from the regression. Column 3 thus confirms the specification of volatility expectations that we use in equation (12).

Overall, figure 3 and table 2 show that market-implied volatility $ev$ contains by itself a large fraction of the total expectation of future volatility; this will allow us to identify the effects of volatility expectations on the real economy.

5.2 Decomposing the variance of realized volatility

The results above show that model-free implied volatility can forecast future realized volatility. The next natural question is how much news there actually is about future volatility, and to what extent fluctuations in realized volatility are surprises. Exploiting properties of expectations, the variance of $rv_t$ can be decomposed into three components,

$$V(rv_t) = V(rv_t - E_{t-1}[rv_t]) + V(E_{t-1}[rv_t] - E_{t-6}[rv_t]) + V(E_{t-6}[rv_t])$$

(19)

where $V(X)$ denotes the unconditional variance of a variable $X$. The first term is the variance of the surprise in $rv$ conditional on information available in the previous period. The second component is the news about $rv_t$ that occurs between months $t - 6$ and $t - 1$. That is, it is the variance of the innovations in the expectation of $rv_t$ over those months. It thus measures how much investors learn about $rv_t$ on average in the five months before it is realized. Finally, $V(E_{t-6}[rv_t])$ is the variance of expectations of $rv$ six months ahead.

To implement this decomposition, we must construct expectations of future volatility at 1- and 6-month horizons. In particular, to account for the possibility that risk premia vary, to measure $E_t[rv_{t+n}]$, we project $rv_{t+n}$ on $ev_{1,t}$ and $ev_{6,t}$ in an unrestricted regression (we obtain similar results if we add other predictors to the forecasting regression, consistent with the view that option prices contain all or nearly all available information about future volatility).

Panel C of Table 1 reports results for the variance decomposition. The table shows that approximately 45 percent of the variance of $rv$ is due to the purely unexpected component, $rv_t - E_{t-1}rv_t$. That is, almost half of the entire variance of $rv_t$ is a surprise (quantitatively similar results hold for alternative specifications of expectations).

About 33 percent of the variance of $rv$ is instead due to the news that investors gain between months $t - 6$ and $t - 1$. Finally, the variance of $E_{t-6}[rv_t]$ accounts for 22 percent of the total variance of $rv_t$.

The results in Table 1 show that when modeling realized variance, it is important to take explicitly into account the fact that almost half of the variation in $rv$ is unpredictable, and only about 20 percent of it is predictable at horizons 6 months or longer. That is, there appears to be
a high-frequency term, like a low-order moving average, that must be accounted for.

6 Variance shocks and the real economy

We now examine the relationship between shocks to volatility and the real economy and estimate the VAR described in the theoretical analysis.

6.1 Data

We focus on monthly data to maximize statistical power, especially since fluctuations in both expected and realized volatility are rather short-lived. We confirm that our results are highly similar in quarterly data, though.

We measure real activity using the Federal Reserve’s measure of industrial production for the manufacturing sector. Employment and hours worked are measured as those of the total private non-farm economy. Inflation is measured using the CPI.

All of the variables except volatility are non-stationary, so we detrend them with a one-sided HP filter with a relatively high smoothing parameter of $1.296 \times 10^7$.

As we discuss further below, the results are robust to alternative specifications of the data. In particular, they are highly similar when we set the HP smoothing parameter to $1.296 \times 10^5$ or drop it entirely; examine volatility in levels or logs; use the 6-month, 3-month, or 1-month market expectation for volatility; or use alternative methods of calculating market volatility expectations (at-the-money implied volatility, variance swaps, or Martin’s (2015) simple variance swap measure).

6.2 Cross-correlations

We begin by examining the raw correlations between measures of real activity and leads and lags of changes in realized and expected volatility.

To obtain innovations, we estimate a bivariate VAR with $rv$ and $ev_6$ as state variables. We then regress four macroeconomic variables – industrial production, hours worked per employee, employment, and inflation – on 12 leads and lags of the two residuals from the VAR. The residuals are not orthogonalized to each other – the regression coefficients simply represent the conditional covariance of the macroeconomic variables with leads and lags of innovations to $rv$ and $ev_6$. Specifically, denoting the residuals from the VAR as $\varepsilon_{rv,t}$ and $\varepsilon_{6,t}$, we estimate regressions of the form

$$Y_t = \sum_{j=-12}^{12} (b_{rv,j}\varepsilon_{rv,t-j} + b_{6,j}\varepsilon_{6,t-j}) + \mu_t \tag{20}$$

where $b_{rv,j}$ and $b_{6,j}$ are coefficients and $\mu_t$ is a residual. $Y_t$ here denotes any of the four macroeconomic series used. We rescale both the residuals and the macroeconomic variables to have unit standard deviations. Figure 4 reports the coefficients, $b_{rv,j}$ and $b_{6,j}$. 

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The figure shows a very consistent pattern. For all macroeconomic variables except the CPI, an increase in $\varepsilon_{rv,t}$ is followed by a significant decline in activity within the next year. Furthermore, $\varepsilon_{rv,t}$ is weakly positively predicted by positive lagged economic conditions. That is, a strong economy is associated with low realized volatility in the past and higher volatility in the future. On the other hand, $\varepsilon_{6,t}$ does not predict declines in activity, and if anything predicts expansions. That is, increases in expected volatility appear to be associated with high future output and employment. The relationship between the innovations and the CPI can be viewed as something of a placebo test. There is little reason to expect that volatility shocks would have substantial effects on inflation, so it is good to see that the estimated coefficients for inflation are close to zero.

While these graphs only reveal cross-correlations between different types of volatility shocks and the macroeconomy, and have no direct causal interpretation, they indicate that there is a substantive difference between increases in realized and expected volatility.

### 6.3 Vector autoregressions

We now examine our main vector autoregressions (VARs) to measure the impact of shocks to realized and expected volatility on the economy. For all the VARs that we run, we include four lags, as suggested by the Akaike information criterion for our main specification.

Following the implications of the model in section 3, our main analysis identifies structural shocks in the VAR using a Cholesky factorization with realized volatility ordered first and expected volatility second. So what we refer to as an identified “realized volatility shock” is simply the residual from a regression of realized volatility on the lagged variables in the VAR. Under the model, this identifies the joint jumps in equity returns and output. As in the model, since expected volatility is ordered second, “expected volatility shocks” here represent changes in volatility expectations that are orthogonal to realized volatility – i.e. separate from ARCH effects. Again, we do not interpret the Cholesky ordering here as a statement about the timing of the shocks. Rather, we interpret the volatility expectations shock as what it literally is in an econometric sense: the change in six-month volatility expectations that cannot be explained by news about realized volatility in the same period.\(^{17}\)

#### 6.3.1 Coefficient estimates

We begin by examining the sums of the coefficients on lagged values of $rv$ and $ev6$ in the VAR. These results are useful to understand because they turn out to drive our key results and they are

\(^{17}\)Christiano, Motto, and Rostagno (2014) study a model with a data-generating process that gives a formal justification for the ordering we use. Suppose realized volatility follows the process

$$rv_t = \phi rv_{t-1} + \varepsilon_{0,t} + \varepsilon_{1,t-1}$$

where $\varepsilon_{1,t}$ is observable to agents on date $t$. Investors’ expectation of volatility at date $t+1$ is thus $E_t rv_{t+1} = \phi rv_t + \varepsilon_{1,t}$. In period $t$, the innovation in $rv_t$ is $\varepsilon_{0,t}$, while the innovation in $E_t rv_{t+1}$ is $\phi \varepsilon_{0,t} + \varepsilon_{1,t}$. If those innovations are rotated with a Cholesky factorization in which $rv$ is ordered first, the identified shock to $rv$ is $\varepsilon_{0,t}$ and the identified shock to $E_t rv_{t+1}$ is $\varepsilon_{1,t+1}$. 

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not dependent on any SVAR identification. Table 3 reports the sum of the coefficients for a range of different VARs. The first row reports the coefficients from the regression of log employment on \( rv \) and \( ev_6 \) from a VAR that includes only those three variables. The sum of the coefficients on \( rv \) is negative, while the sum of the coefficients on \( ev_6 \) is actually positive. High levels of realized volatility forecast low employment in the future, but high levels of expected volatility actually forecast high employment. As is often found with VARs, the coefficients are not statistically significant, and their difference only has a p-value of 0.14. Nevertheless, this basic result will appear consistently through our results and will drive the other results we report below (which will themselves have more statistical power).

The second row of table 3 replaces log employment with log industrial production and finds similar though statistically weaker results. The third and fourth rows report the coefficients on employment and industrial production from our main VAR that includes those two variables, \( rv \), \( ev_6 \), and the Fed funds rate. Finally, the right-hand panel of table 3 reports results from the 1988–2006 subsample that eliminates the two biggest jumps in realized volatility. In all cases, we find similar results.

### 6.3.2 Impulse responses

We now examine impulse response functions, which describe the full dynamic response of the variables in the economy to each of the identified shocks.

We begin by estimating a version of the VAR that has been used in the past literature that does not control for realized volatility. Here the VAR only includes one-month implied volatility, \( ev_1 \).\(^{18}\) All IRFs that we report are in response to unit standard deviation shocks. In figure 5, the shock to volatility expectations has a half-life of approximately 8 months and reduces employment and industrial production by statistically and economically significant amounts. The peak reduction in both following a volatility shock is approximately 0.35 percent. The magnitude of these responses is in line with those obtained by Basu and Bundick (2015), for example, though slightly larger. As noted above, by only looking at one volatility indicator (in this case \( ev_1 \)), it is impossible to distinguish whether the macroeconomic effects are related to, the component of realized volatility with which \( ev_1 \) is correlated or the expectations component.

Figure 6 presents our main VAR results. The figure has four columns for the responses of \( rv \), \( ev_6 \), employment, and industrial production to the shocks. We measure responses to unit standard deviation shocks to both realized and expected volatility.

The first row of figure 6 shows that a shock to realized volatility is highly transitory: the IRF falls by half within two months, and by three-fourths within 5 months (compared to the eight-month half-life of shocks to the VXO). Consistent with the results in section 5, then, realized volatility appears to have a highly transitory component. Naturally, the shock to realized volatility also

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\(^{18}\)We use the VXO for this analysis, which is the equivalent of the VIX for the S&P 100 rather than the S&P 500, since it has a longer time series than the VIX; this is why previous literature has focused on the VXO rather than the VIX. In practice, the same results hold with VIX and VXO.
affects volatility expectations – the dynamics of 6-month volatility expectations appear to line up reasonably well with what is implied by the IRF for realized volatility itself.

As to the real economy, a unit standard deviation increase in realized volatility is associated with statistically and economically significant declines in both employment and industrial production, reducing them both by 0.3 percent. In terms of the model, this is consistent with the view that jumps in the stock market (which are typically negative) are associated with declines in output. That is, \( J < 0 \) and \( k_{Y,J} > 0 \).

The effects of a shock to volatility expectations are much different. First, as we would expect, a shock to volatility expectations forecasts high realized volatility in the future at a high level of statistical significance. That result alone is important: it says that shocks to ev\(_6\), after controlling for the current shock to rv, are able to significantly predict future volatility. That is, option prices contain information about future volatility even after controlling for current and past realized volatility. Surprisingly, though, increases in volatility expectations are associated with no significant change in employment or industrial production. Employment declines only trivially, and industrial production appears to actually increase.

The bottom row of panels in figure 6 reports the difference in the IRFs for the rv and ev\(_6\) shocks. The first two panels show that the two shocks have highly similar impacts on the future paths of realized and expected volatility (though on impact they obviously have different effects on rv simply due to the ordering of the identification). The cumulative effect of the two shocks on future rv (ignoring the impact period) is 0.76 for the rv shock and 0.87 for the ev\(_6\) shock. In other words, the two shocks have highly similar effects on cumulative future volatility. Similarly, the rv shock raises future volatility expectations (i.e. the sum of the second panel in the top row) by 0.80, while the ev\(_6\) shock raises them by 1.13.

The two shocks thus have approximately equal effects on future uncertainty – if anything, the ev shock has bigger effects on future uncertainty. But the figure shows that the two shocks in fact have significantly different effects on employment and output. rv has much more negative effects, and that difference is statistically significant.

So the rv and ev\(_6\) shocks have similar effects on future uncertainty but markedly different effects on the economy. What can explain that difference? In terms of the VAR, the only qualitative difference between the two shocks is that the rv shock has a large effect on realized volatility on impact. It is this initial impact effect that seems to drive output down. The results in figure 6 therefore show that periods of high realized volatility are associated with declines in activity, but shocks to volatility expectations have no significant effect on the economy.
6.3.3 Forecast error variance decompositions

To further understand the importance of the \( rv \) and \( ev \) shocks, figure 7 reports forecast error variance decompositions. As in figure 6, we report the effect of the \( rv \) shock, the \( ev \) shock, and their difference. The realized variance shock explains 10 percent of the variance of employment and 5 percent of the variance of industrial production at most horizons, while the point estimates for the fraction of the variance accounted for by expected volatility are close to zero. The upper end of the 95-percent confidence interval for the \( ev \) shock is below 5 percent for the first 10 months. The upper end of the 95-percent confidence interval for the \( rv \) shock, though, reaches as high as 25 percent for employment and 20 percent for industrial production 10 months ahead, indicating that \( RV \) can potentially be an important driver of the real economy (though this is not a causal statement – our model says that this can be simply due to common shocks to \( RV \) and output).

The bottom row of figure 6 shows that the fraction of the forecast error of volatility expectations accounted for by the two shocks is not significantly different, again emphasizing that they have similar effects on volatility expectations – the main difference between the two shocks is that one is affecting realized variance on impact, and the other is not. We see, though, that the difference in the fraction of the forecast error of employment and industrial production explained by the two shocks is meaningfully different economically, and marginally significantly at shorter horizons.

So in terms of the model, the estimated IRFs imply \( k_{Y,\sigma} \), the response of real activity to uncertainty, is zero or potentially even positive. The variance decompositions furthermore show that shocks to expected volatility have little or no importance for economic outcomes. Shocks to volatility expectations, after controlling for realized volatility and ARCH effects, seem to be associated with little change in the state of the real economy.

6.3.4 Quarterly data

In order to examine the effects of our two shocks on a wider range of variables, we also estimate a VAR using quarterly data similar to that of Basu and Bundick (2015) that includes, in addition to the two volatility series, GDP, consumption, investment, hours, the GDP deflator, the M2 money supply, and the Fed Funds rate (using the Wu and Xia (2014) shadow rate when the zero lower bound binds). Figure A.6 shows that following an increase in realized volatility, we obtain the same comovement emphasized by Basu and Bundick (2015): output, consumption, investment, and hours worked all decline, all statistically significantly. As we would expect, investment is most sensitive to the shock to realized volatility, with a peak response four times larger than that of output and six times larger than that of consumption. For expected volatility we again find no statistically or economically significant effects, with small initial declines and subsequent rebounds. Furthermore, the magnitude of the point estimates for the declines following expected volatility shocks is again not only statistically insignificant but also far smaller than the declines in response to realized volatility.

To summarize, then, we confirm the usual result that increases in one-month stock market
volatility expectations are contractionary when they are included alone. But when the VAR includes both realized and expected volatility, we find that it is the increase in realized volatility that is associated with contractions, while increases in expected volatility are weakly associated with expansions.

6.4 Estimation through local projections

In order to ensure that our results are not driven by the particular structure of the VAR that we examine, we now estimate impulse responses using the nonparametric method of Jordà (2005). The Jordà method uses local projections that estimate impulse responses essentially as partial derivatives. They rely much less strongly on the internal propagation of a VAR. Specifically, the Jordà projection estimates the partial derivative of the expected value of a variable, $E_t Y_{t+j}$, with respect to the current values of the state variables, $X_t$. That is, it yields $\partial E_t Y_{t+j} / \partial X_t$, which is exactly the definition of an impulse response function in a VAR, but the Jordà method is far less reliant on the strict VAR structure. In particular, the IRF at horizon $j$ is obtained through direct regressions of $Y_{t+j}$ on $X_t$ and its lags. The difference between the Jordà projection and the VAR IRF is therefore equivalent to the difference between a direct and an iterated forecast (see Marcellino, Stock, and Watson (2006)).

Figure 8 reports estimates and confidence intervals for IRFs using the Jordà (2005) projection method. The red dotted lines indicate the IRFs from figure 6. Across all nine panels in the figure, the local projections, even though they have none of the internal propagation of the VAR, yield IRFs that are highly similar to what we obtained through OLS estimation of the VAR. Increases in $rv$ are associated with declines in employment and industrial production, while increases in $ev_6$ are not, and the difference in the IRFs for the two variables is again statistically significant. The results in figure 8 thus imply that there is nothing about the VAR itself – i.e. some sort of strange feedback between the endogenous variables – that is driving our results.

6.5 Robustness tests

We examine a range of perturbations of our main specification from figure 6. First, we consider alternative orderings of the variables in the VAR. The effects of the ordering depend ultimately on the correlation matrix of the innovations, which we report below:

<table>
<thead>
<tr>
<th></th>
<th>rv</th>
<th>ev_6</th>
<th>Fed Funds</th>
<th>Empl.</th>
<th>IP</th>
</tr>
</thead>
<tbody>
<tr>
<td>rv</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ev_6</td>
<td>0.66</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fed Funds</td>
<td>-0.02</td>
<td>-0.07</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Empl.</td>
<td>0.04</td>
<td>0.10</td>
<td>0.03</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>IP</td>
<td>-0.01</td>
<td>0.10</td>
<td>0.03</td>
<td>0.55</td>
<td>1</td>
</tr>
</tbody>
</table>

Consistent with standard models of equity volatility (e.g. ARCH and GARCH models), the shocks to realized and expected volatility are correlated, but far from collinear. Their ordering
in the VAR is therefore obviously relevant for the results. However, their innovations are almost completely uncorrelated with those to the other variables, implying that the relative ordering of the financial and macro variables is unlikely to affect the results. We confirm that intuition in the appendix.

Figure A.5 reports results from a monthly VAR analogous to that of figure 6 where we reverse the ordering of realized and expected volatility in the Cholesky decomposition. In this decomposition, the first shock represents a shock to both expected volatility and realized volatility; the second shock is a shock to realized volatility that has no effect on volatility expectations – i.e. a purely transitory shock. The figure shows that both shocks have essentially the same effect on employment and industrial production. That result is entirely consistent with our main analysis: it shows that when the two shocks differ in whether they have any effect on volatility expectations, they still have the same impact on the real economy. So the volatility expectations component does not affect the IRFs.

Figures A.7 to A.10 in the appendix report a range of additional robustness tests. Figures A.7 and A.8 show the response of log employment to the two volatility shocks, and figures A.9 and A.10 report the response of log industrial production. In each figure, the rows correspond to different specifications of the estimation. The left panels report responses to unit standard deviation shocks to \( rv \), while the middle panels report the responses to unit standard deviation shocks to volatility expectations, \( ev_6 \). The right panel reports the difference between the two impulse-response functions. The figures examine eight robustness tests:

1. No detrending
2. Quarterly instead of monthly data.
4. Ordering \( rv \) and \( ev_6 \) last in the Cholesky decomposition.
5. Calculating volatility expectations using 6-month at-the-money Black–Scholes option implied volatility instead of the 6-month VIX.
6. Replacing \( ev_6 \) with \( ev_1 \), the one-month VIX.
7. Controlling for the level of the S&P 500 first in the VAR.
8. Using \( RV \) and \( EV_6 \) (i.e. levels rather than logs).

The results of the robustness tests are qualitatively and quantitatively consistent with our baseline results.

Finally, we also find similar results in unreported additional robustness tests: using the \( ev_3 \); using the SVIX of Martin (2015), as an alternative measure of expected volatility; using different detrending parameters when HP-filtering the series; not detrending the volatility variables only (since they are stationary in levels); using variance swap data instead of option data to construct \( ev_6 \) (for the period 1996-2013); and using the 12-month variance swap to construct \( ev_{12} \) (for the period 1996-2013).
7 Equilibrium model and further evidence

The empirical evidence presented thus far is consistent with the view that there are periodic discrete jumps in activity that induce a negative relationship between realized volatility and output: investors appear to demand much larger compensation for exposure to shocks to realized than expected volatility, and realized volatility is associated with significantly larger declines in output than expected volatility is. In this section we present direct evidence on skewness in the economy and also present a simple structural equilibrium model that is consistent with the broad features of the data – skewness, risk premia, and VAR results – and that also shows that our VAR identification scheme can isolate the underlying economic shocks.

7.1 Skewness

The empirical model presented in Section 3 suggests that a potential source of negative correlation between output and realized volatility is negatively skewed shocks. Specifically, if some shock \( \varepsilon \) is negatively skewed, then \( E[\varepsilon^3] < 0 \Rightarrow \text{cov}(\varepsilon, \varepsilon^2) < 0 \). That is, negative skewness implies a negative correlation between \( \varepsilon^2 \) and \( \varepsilon \) itself. So high realized volatility (\( \varepsilon^2 \)) should be associated with downturns. The obvious question, then, is whether shocks to output and asset returns are actually skewed to the left. There are large literatures studying skewness in both aggregate stock returns and economic growth. We therefore provide just a brief overview of the literature and the basic evidence.

Table 4 reports the skewness of monthly and quarterly changes in a range of measures of economic activity. Nearly all the variables that we examine are negatively skewed, at both the monthly and quarterly levels. One major exception is monthly growth in industrial production, but that result appears to be due to some large fluctuations in the 1950's. When the sample is cut off at 1960, the results for industrial production are consistent with those for other variables.

In addition to real variables, table 4 also reports realized and option-implied skewness for S&P 500 returns.\(^\text{19}\) The implied and realized skewness of monthly stock returns is substantially negative, and in fact surprisingly similar to the skewness of capacity utilization. The realized skewness of stock returns is less negative than option-implied skewness, which is consistent with investors demanding a risk premium on assets that have negative returns in periods when realized skewness is especially negative (i.e. that covary positively with skewness).

In addition to the basic evidence reported here, there is a large literature providing much more sophisticated analyses of asymmetries in the distributions of output and stock returns. Morley and Piger (2012) provide an extensive analysis of asymmetries in the business cycle and review the large literature. They estimate a wide range of models, including symmetrical ARMA specifications, regime-switching models, and frameworks that allow nonlinearity. The models that fit aggregate output best have explicit non-linearity and negative skewness. Even after averaging across models

\(^{19}\)We obtain option-implied skewness from the CBOE's time series of its SKEW index, which is defined as \( SKEW = 100 - 10 \times \text{Skew}(R) \). We thus report \( 10 - SKEW/10 \).
using a measure of posterior probability, which puts substantial weight on purely symmetrical models, Morley and Piger find that their measure of the business cycle is substantially skewed to the left, consistent with the results reported in table 4. More recently, Salgado, Guvenen, and Bloom (2016) provide evidence that left skewness is a robust feature of business cycles, at both the macro and micro levels and across many countries.

The finance literature has also long recognized that there is skewness in aggregate equity returns and in option-implied return distributions (see Campbell and Hentschel (1992), Ait-Sahalia and Lo (1998), and Bakshi, Kapadia, and Madan (2003), for recent analyses and reviews). The skewness that we measure here appears to be pervasive and has existed in returns reaching back even to the 19th century (Campbell and Hentschel (1992)).

Taken as a whole, then, across a range of data sources and estimation methods, there is a substantial body of evidence that fluctuations in the economy are negatively skewed. In a world of negative skewness, it is not surprising that measures of realized volatility are correlated with declines in activity, simply because skewness is related to the third moment: $E[\varepsilon^3] = E[\varepsilon \cdot \varepsilon^2]$.

7.2 An equilibrium model

The empirical evidence presented thus far is consistent with the view that shocks to aggregate realized volatility have a large effect on the macroeconomy while shocks to expected aggregate volatility do not. In this section we present a stylized equilibrium model that is both consistent with our evidence and close to the workhorse RBC model. We deliberately keep the model simple in order to highlight the economic channels that are at work, but despite the simplicity the model is consistent with a wide variety of real and asset pricing facts.

Our model is an RBC model where aggregate TFP growth is heteroskedastic and skewed to the left. We want it to be consistent with the three facts presented in this paper: (1) Shocks to realized volatility are associated with declines in real activity, while shocks to expected volatility are not; (2) Sharpe ratios on short-term claims to volatility are much more negative than those on longer-term claims; and (3) output growth and equity returns are negatively skewed.

7.2.1 Model structure

The model is meant to represent the simplest possible equilibrium production model consistent with the qualitative features of our data. Output is produced with technology $A_t$ and capital $K_t$

$$Y_t = A_t K_{t-1}^{\alpha}$$

(22)

(this can be thought of as a Cobb–Douglas production function with constant labor normalized to 1). We set $\alpha = 0.33$, consistent with capital’s share of income. Capital is produced subject to
adjustment costs according to the production function

\[ K_t = (1 - \delta) K_{t-1} + K_{t-1} \left( \frac{I_t}{K_{t-1}} - \frac{\zeta}{2} \left( \frac{I_t}{K_{t-1}} - \frac{I_t}{I/K} \right)^2 \right) \]  

(23)

where \( I_t \) is gross investment, \( \zeta \) is a parameter determining the magnitude of adjustment costs and \( I/K \) is the steady-state investment/capital ratio. We set \( \delta = 0.08/12 \) (corresponding to a monthly calibration) and \( \zeta = 1 \).\(^{20}\) Given the structure for adjustment costs and production, the equilibrium price and return on a unit of installed capital are

\[ P_{K,t} = \frac{1}{1 - \zeta \left( \frac{I_t}{K_{t-1}} - 1 \right)} \]

(24)

\[ R_{K,t} = \frac{\alpha A_t K_{t-1}^{\alpha-1} + (1 - \delta) P_{K,t}}{P_{K,t-1}} \]

(25)

We assume there is a representative agent who maximizes expected utility,

\[ \max_{C} \sum_{j=0}^{\infty} \beta^j \frac{C_{t+j}^{-\rho}}{1 - \rho} \]

(26)

subject to the budget constraint

\[ C_t + I_t \leq Y_t \]

(27)

We set risk aversion, \( \rho \), to 10, in order to generate moderately large risk premia (this will cause the model to have unrealistic interest rate implications, but that is not our focus). \( \beta \) is set to \( 0.99^{1/12} \).

The model is closed by the Euler equation,

\[ 1 = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-1} R_{K,t+1} \right] \]

(28)

We model realized volatility as the squared excess return on capital,

\[ RV_t^2 = (R_{K,t} - R_{f,t})^2 \]

(29)

where \( R_{f,t} \) is the risk-free rate, with \( 1 = E_t \left[ \beta (C_{t+1}/C_t)^{-1} R_{f,t} \right] \). It is then straightforward to construct prices on claims to future realized volatility. The price of a claim to \( RV_{t+j}^2 \) on date \( t \) is denoted \( P_{V,j,t} \).

\(^{20}\)See, e.g., Cummins, Hassett, and Hubbard (1994) for estimates of adjustment costs similar to this value. \( \zeta = 1 \) is on the lower end of estimates based on aggregate data and more consistent with micro evidence, but our results are not sensitive to the choice of this parameter.
The only exogenous variable in the model is technology, $A_t$. We assume it follows the process

$$
\Delta \log A_t = \sigma_{t-1} \varepsilon_t - J (\nu_t - \bar{p} \sigma_{t-1}) + \mu 
$$

(30)

$$
\log \sigma_t = \phi_{\sigma} \log \sigma_{t-1} + \sigma_{\sigma} \eta_t + \kappa_{\sigma,A} \Delta \log A_t 
$$

(31)

$$
\varepsilon_t, \eta_t \sim N(0,1) 
$$

(32)

$$
\nu_t \sim \text{Bernoulli}(\bar{p} \sigma_{t-1}) 
$$

(33)

Technology follows a random walk in logs with drift $\mu$, set to 2 percent per year. $\varepsilon_t$ is a normally distributed innovation that affects technology in each period, while $\nu_t$ is a shock that is equal to zero in most periods but equal to 1 with probability $\bar{p} \sigma_{t-1}$ – that is, it induces downward jumps in technology, with $J$ determining the size of the jump (or disaster) and $\bar{p}$ the average frequency. $\sigma_t$ determines the volatility of shocks to technology. It is driven by two shocks: an independent shock $\eta_t$ and also the innovations to technology in period $t$. A positive technology shock may feed into lower volatility in the future. The volatility process thus has two features that will be important in matching the data: it has news shocks, and it is countercyclical (for $\kappa_{\sigma,A} < 0$).

$\phi_{\sigma}$ is set so that shocks to volatility have a half-life of 6 months, consistent with the behavior of the VIX. $\kappa_{\sigma,A}$ is set to -5, which implies that a jump in technology, $J \nu_t$, increases $\sigma_t$ by half of one standard deviation. $\bar{\sigma}_x$ is set so that normally distributed shocks on average generate a standard deviation of technology growth of 1 percent per year (consistent with consumption behavior). Jumps on average reduce technology by 5 percent and are calibrated to occur once every 10 years on average. We thus think of them as representing small disasters or relatively large recessions (consistent Backus, Chernov, and Martin (2011) and with the view of skewed recessions in Salgado, Guvenen, and Bloom (2016)), rather than depression-type disasters.\textsuperscript{21}

We solve the model by projecting the decision rule for consumption on a set of Chebyshev polynomials up to the 5th order (a so-called global solution) to ensure accuracy not only for real dynamics but also for asset prices and realized volatility.

### 7.2.2 Simulation results

We examine three sets of implications of the model: VAR estimates, risk premia, and skewness. All results are population statistics calculated from a simulation lasting 13,000 years.

Consistent with the empirical results reported above, all major aggregate variables in the economy – growth in output, consumption and investment, and stock returns – are skewed to the left. Growth rates of output, consumption, and investment have skewness of approximately -3.3 (inheriting the skewness of $\Delta \log A_t$). Excess returns on capital have skewness of -2.8. We thus obtain the left skewness observed in Table 4, but the model has more skewness than is observed empirically due to the relatively large jumps in the calibration.

\textsuperscript{21}A realistic extension of the model would be to allow for jumps to be drawn from a distribution, rather than all having the same size. See, e.g., Barro and Jin (2011).
Figure 9 plots the Sharpe ratios of volatility claims in the model that correspond to the forward volatility claims examined in Figure 1. As in Figure 1, the Sharpe ratio of the one-month asset, which is a claim to realized volatility, is far more negative than the Sharpe ratios for the claims with longer maturities. Intuitively, this is because shocks to volatility expectations, $\eta_t$, have relatively small effects on consumption, hence earning a small risk premium. Shocks to realized volatility, on the other hand, tend to isolate the jumps, $\nu_t$, (as we will show below), so they earn larger premia.

Finally, the solid lines in figure 10 summarize the results of VARs estimated from simulations of the model. The VAR in the simulations replicates the one used in the main analysis above. In particular, it includes realized volatility, expected volatility ($P_{V,1,t}$), and the level of output, with shocks identified in that order. In figure 10, the realized volatility shock is scaled so that the sum of the response of realized volatility to the expected volatility shock is the same as the sum of the response of realized volatility to its own shock (consistent with the empirical results).

We see, as in the main results, that a unit standard deviation shock to realized volatility has a transitory effect on realized volatility, a persistent effect on expected volatility, and a negative effect on output (of a similar magnitude to what is observed empirically). The “expected volatility shock”, which, as above, is the innovation to $P_{V,1,t}$ controlling for current realized volatility and lags, has a predictive effect on future realized volatility, but only a quantitatively trivial effect on output. The bottom row shows the difference in the IRFs, and we see that the RV shock has substantially more negative effects on output than the EV shock has.

Because the VAR is on simulated data, it is possible to compare the shocks identified by the VAR to the true structural shocks in the model. The correlation matrix of the six shocks is reported below.

<table>
<thead>
<tr>
<th>Structural shocks</th>
<th>$J_{\nu_t}$</th>
<th>$\eta_t$</th>
<th>$\varepsilon_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VAR identified RV shock</td>
<td>0.98</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>shocks</td>
<td>0.00</td>
<td>0.94</td>
<td>-0.06</td>
</tr>
<tr>
<td>Output shock</td>
<td>0.07</td>
<td>0.07</td>
<td>0.93</td>
</tr>
</tbody>
</table>

The RV shock is correlated nearly exclusively with $J_{\nu_t}$, the jump shock in the model. So the VAR successfully identifies the jumps as realized volatility shocks, which are then structurally (though not causally) related to declines in output. The EV shock, as we would hope, is, similarly, almost purely correlated with $\eta_t$, the volatility news shock. Finally, the output shock from the VAR is primarily correlated with $\varepsilon_t$, the small shock to technology. So our main VAR specification does a good job in this setting – a non-linear production model – of actually identifying true structural shocks and also fitting the qualitative behavior of our empirical VAR analysis.

The fact that the shocks identified by the VAR are very similar to the structural shocks in the model suggests that the impulse responses estimated by the VAR should be very similar to the effects of the shocks in the structural model itself. Figure 10 therefore plots, in addition to the IRFs estimated from the simulation, the responses of realized volatility, expected volatility, and output, to shocks to $J_{\nu_t}$ and $\eta_t$ – the structural jump shock and volatility news shock, respectively. The
structural IRFs are the dashed lines. They are scaled so that the response of expected volatility to the structural shocks is the same as that in the simulated VAR.

We see that the response of realized and expected volatility to the VAR-estimated RV and EV shocks are nearly identical to the responses to the true structural shocks $J_{\nu t}$ and $\eta_t$ (due to the normalization to match the EV IRFs, the similarity there is only notable for the shape, rather than the level). Most importantly, the response of output to the estimated shocks is rather similar to the response to the structural shocks. Output falls by 0.2 percent following the estimated RV shock, while it falls by 0.3 percent following the structural shock. The second row shows that there is essentially zero response to both the estimated EV shock and to $\eta_t$, even though both do increase uncertainty and realized volatility.

This section thus shows that a simple production model can match the basic features of the data that we have estimated in this paper: there is a large risk premium for realized but not expected volatility, output responds negatively to shocks to realized volatility, but not to shocks to expected volatility (after controlling for contemporaneous realized volatility), and economic activity and stock returns are both skewed to the left.

8 Conclusion

The goal of this paper is to understand whether shocks to uncertainty in financial markets, as proxied for by option-implied volatility, have negative effects on the economy. We find that shocks to expected volatility in the future, after controlling for current realized volatility, do not have meaningful negative effects on the economy. The evidence we present favors the view that bad times are volatile times, not that volatility causes bad times. A leading hypothesized explanation for the slow recovery from the 2008 financial crisis has been that uncertainty since then has been high. Our evidence suggests that uncertainty may not have been the driving force, and that economists should search elsewhere for an explanation to the slow recovery puzzle.

More constructively, this paper aims to lay out a specific view of the joint behavior of stock market volatility and the real economy. There appear to be negative shocks to the stock market that occur at business cycle frequencies, are associated with high realized volatility and declines in output, and are priced strongly by investors. The simple idea that fundamentals are skewed left can explain our VAR evidence, the pricing of volatility risk, and the negative unconditional correlation between economic activity and volatility.

References


Figure 1: Forward variance claims: returns and prices

Note: Panel A shows the annualized Sharpe ratio for the forward variance claims, constructed using Variance Swaps. The returns are calculated assuming that the investment in an n-month variance claim is rolled over each month. Dotted lines represent 95% confidence intervals. All tests for the difference in Sharpe ratio between the 1-month variance swap and any other maturity confirm that they are statistically different with a p-value of 0.03 (for the second month) and < 0.01 (for all other maturities). The sample used is 1996-2013. For more information on the data sources, see Dew-Becker et al. (2015). Panel B shows the average prices of forward variance claims of different maturity, constructed from option prices, for the period 1983–2014. All prices are reported in annualized volatility terms. Maturity zero corresponds to average realized volatility.

Figure 2: Time series of realized variance and expectations

Note: Time series of realized variance ($RV$), and 6-month expectations ($EV_6$), in annualized units. Grey bars indicate NBER recessions.
Figure 3: Predictive regressions

Note: Coefficients of regressions of $rv$ on lagged $rv$ and expected volatility, $ev_6$, at different lags (X axis). Top row reports coefficient of univariate regressions of $rv$ on lagged $ev_6$, for different lags. Bottom row reports coefficients of multivariate regressions of $rv$ onto lagged $rv$ and lagged $ev_6$, for different lags (respectively, left and right panel).
Figure 4: Correlation of macro variables to \( rv \) and expected volatility shocks

**Note:** The figure shows the response of macroeconomic variables to shocks to \( rv \) (left) and expected volatility, \( ev_6 \) (right), before and after the shock (lag 0 on the X axis). In particular, the solid line reports the coefficient of regressions of macroeconomic variables (Industrial Production, first row, Hours Worked, second row, Employment, third row, and Inflation, fourth row) on leads and lags of shocks to \( rv \) (left column) and expected volatility \( ev_6 \) (right column). Dashed lines indicate 90% confidence intervals, dotted lines 90% confidence intervals. The sample covers 1983 to 2014.
Note: The figure shows impulse response functions (with 90% and 95% CI) of volatility (measured by VXO), employment and industrial production to a shock to VXO, in a VAR with VXO, federal funds rate, log employment and log industrial production. Impulse response functions reported correspond to structural shocks with Choleski decomposition (in the order reported above). Sample covers 1986-2014. All macroeconomic series were detrended with a one-sided HP filter.
Note: The figure shows impulse response functions (with 90% and 95% CI) of $rv$, volatility expectations ($ev_6$), employment and industrial production to shocks to $rv$ and $ev_6$, in a VAR with $rv$, $ev_6$, federal funds rate, log employment and log industrial production. Impulse response functions reported correspond to structural shocks with Choleski decomposition (in the order reported above). First row shows the response to a one-standard-deviation shock to $rv$. Second row shows the response to a one-standard-deviation shock to $ev$. Third row shows the difference in the response to the two shocks. Sample covers 1983-2014. All macroeconomic series were detrended with a one-sided HP filter.
Figure 7: Forecast error variance decomposition

Note: The figure shows the fraction of the forecast error variance (FEV) of $rv$, volatility expectations ($ev_6$), employment and industrial production to shocks to $rv$ and $ev_6$, in the VAR of figure 6. The figure also reports 90% and 95% confidence intervals. First row shows the fraction of the FEV due to the $rv$ shock. Second row shows the fraction of FEV due to the $ev$ shock. Third row shows the difference in the fractions of FEV due to the $rv$ and the $ev$ shock.
Note: The figure shows impulse response functions from the local projections (Jorda 2005), with 90% and 95% CI, of rv, volatility expectations (ev6), employment and industrial production to shocks to rv and ev6, in a VAR with rv, ev6, federal funds rate, log employment and log industrial production. The responses reported correspond to structural shocks with Choleski decomposition (in the order reported above). First row shows the response to a one-standard-deviation shock to rv. Second row shows the response to a one-standard-deviation shock to ev. Third row shows the difference in the response to the two shocks. The figure also reports the VAR-based impulse-response function for comparison. Sample covers 1983-2014. All macroeconomic series were detrended with a one-sided HP filter.
Figure 9: Annual Sharpe ratios on forward claims (simulated structural model)

Note: The figure shows annual Sharpe ratio on forward variance claims of maturity 1 to 12 months, in the simulated model of section 7. The Sharpe ratios are constructed as in Figure 1.
Figure 10: IRFs from structural model

Note: The figure shows impulse response functions from data simulated from the model in Section 7. Solid lines correspond to IRFs estimated using our VAR methodology as in Figure 6. Dashed lines correspond to IRFs for the two structural shocks $J_{νt}$ and $η_t$. 
Table 1: Descriptive statistics

<table>
<thead>
<tr>
<th>Panel A: Descriptive statistics</th>
<th>Mean</th>
<th>Std.</th>
<th>Skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>RV</td>
<td>15.31</td>
<td>9.34</td>
<td>3.62</td>
</tr>
<tr>
<td>EV$_6$</td>
<td>19.99</td>
<td>6.60</td>
<td>1.36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$rv$</td>
<td>1.00</td>
<td>0.78</td>
<td>0.17</td>
<td>-0.33</td>
<td>-0.20</td>
</tr>
<tr>
<td>$ev_6$</td>
<td>0.78</td>
<td>1.00</td>
<td>0.06</td>
<td>-0.38</td>
<td>-0.14</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.17</td>
<td>0.06</td>
<td>1.00</td>
<td>-0.70</td>
<td>0.10</td>
</tr>
<tr>
<td>Capacity Utilization</td>
<td>-0.33</td>
<td>-0.38</td>
<td>-0.70</td>
<td>1.00</td>
<td>-0.06</td>
</tr>
<tr>
<td>S&amp;P 500 return</td>
<td>-0.20</td>
<td>-0.14</td>
<td>0.10</td>
<td>-0.06</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**Panel C: Decomposition of $V(rv)$**

| Predictor | $V(rv)$ | $V(rv|_t+1 - E^1_t)$ | $V(E^5_{t+5} - E^6_t)$ | $V(E^6_t)$ |
|-----------|---------|----------------------|------------------------|------------|
|           | 48%     | 31%                  | 21%                    |            |

**Note:** The table reports various statistics on realized volatility, forwards and their relationship. Panel A reports the mean, standard deviation and skewness of realized volatility and expected volatility $ev_6$ (the market expectation of realized variance up to 6 months in the future). Panel B reports the correlations between those variables and with macroeconomic and financial variables: unemployment, capacity utilization, and the S&P 500 return. Panel C computes a variance decomposition for the variance of $rv$, $V(rv)$, into the surprise component $V(rv|_t+1 - E^1_t)$, the volatility news 1 to 5 months ahead, $V(E^5_{t+5} - E^6_t)$, and the volatility news 6 months ahead, $V(E^6_t)$. The decomposition is obtained using the 1-month and 6-month market expectation, $ev_1$ and $ev_6$, to predict volatility at the 1-month and 6-month horizon. Sample period is 1983-2014.

Table 2: Predictability of $rv$

<table>
<thead>
<tr>
<th>Predictors</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ev_6$</td>
<td>0.82***</td>
<td>0.45***</td>
<td>0.46***</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.11)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>$rv$</td>
<td>0.32***</td>
<td>0.32***</td>
<td>0.32***</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.09)</td>
<td></td>
</tr>
<tr>
<td>$FFR$</td>
<td>0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Empl.$</td>
<td>-0.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.43)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$IP$</td>
<td>0.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.00)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>377</td>
<td>377</td>
<td>377</td>
</tr>
<tr>
<td>Adj. R2</td>
<td>0.38</td>
<td>0.43</td>
<td>0.42</td>
</tr>
</tbody>
</table>

**Note:** The figure reports the results of linear predictive regressions of 6-month $rv$ on various macroeconomic variables, with Hansen-Hodrick standard errors (6 lags).
Table 3: Cumulative coefficients on $rv$ and $ev$ lags

<table>
<thead>
<tr>
<th></th>
<th>VAR specification</th>
<th>$rv$</th>
<th>$ev$</th>
<th>diff.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Full Sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Empl.</td>
<td>Empl., $rv$, $ev$</td>
<td>-0.11</td>
<td>0.07</td>
<td>-0.18</td>
<td>0.14</td>
</tr>
<tr>
<td>IP</td>
<td>IP, $rv$, $ev$</td>
<td>-0.20</td>
<td>0.14</td>
<td>-0.34</td>
<td>0.39</td>
</tr>
<tr>
<td>Empl.</td>
<td>IP, Empl., $rv$, $ev$</td>
<td>-0.10</td>
<td>0.06</td>
<td>-0.16</td>
<td>0.20</td>
</tr>
<tr>
<td>IP</td>
<td>IP, Empl., $rv$, $ev$</td>
<td>-0.21</td>
<td>0.15</td>
<td>-0.36</td>
<td>0.41</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>VAR specification</th>
<th>$rv$</th>
<th>$ev$</th>
<th>diff.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel B: 1988-2006</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Empl.</td>
<td>Empl., $rv$, $ev$</td>
<td>-0.15</td>
<td>0.13</td>
<td>-0.29</td>
<td>0.16</td>
</tr>
<tr>
<td>IP</td>
<td>IP, $rv$, $ev$</td>
<td>-0.25</td>
<td>0.31</td>
<td>-0.56</td>
<td>0.37</td>
</tr>
<tr>
<td>Empl.</td>
<td>IP, Empl., $rv$, $ev$</td>
<td>-0.16</td>
<td>0.03</td>
<td>-0.19</td>
<td>0.38</td>
</tr>
<tr>
<td>IP</td>
<td>IP, Empl., $rv$, $ev$</td>
<td>-0.11</td>
<td>0.60</td>
<td>-0.71</td>
<td>0.29</td>
</tr>
</tbody>
</table>

**Note:** The figure reports the sum of the coefficients of IP and Employment on the lags of $rv$ and $ev$ in VAR specifications that include $rv$, $ev$, as well as IP or Employment (or both). Columns 1 and 2 report the sums of the coefficients on the lags of $rv$ and $ev$, respectively; column 3 reports the difference in these sums, and column 4 reports p-values for tests of this difference. Panel A performs the analysis on the full sample (1983-2014), while Panel B focuses on the sample 1988-2006.

Table 4: Skewness

<table>
<thead>
<tr>
<th></th>
<th>Monthly</th>
<th>Quarterly</th>
<th>Start of sample (year)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: real economic activity</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employment</td>
<td>-0.41</td>
<td>-0.41</td>
<td>1948</td>
</tr>
<tr>
<td>Capacity Utilization</td>
<td>-1.02</td>
<td>-1.30</td>
<td>1967</td>
</tr>
<tr>
<td>IP</td>
<td>0.17</td>
<td>-0.16</td>
<td>1948</td>
</tr>
<tr>
<td>IP, starting 1960</td>
<td>-0.93</td>
<td>-1.28</td>
<td>1960</td>
</tr>
<tr>
<td>Y</td>
<td>-0.11</td>
<td></td>
<td>1947</td>
</tr>
<tr>
<td>C</td>
<td>-0.28</td>
<td></td>
<td>1947</td>
</tr>
<tr>
<td>I</td>
<td>-0.03</td>
<td></td>
<td>1947</td>
</tr>
</tbody>
</table>

|                      |         |           |                        |
| **Panel B: skewness of S&P 500 monthly returns** |         |           |                        |
| Implied (since 1990) | -1.81   |           |                        |
| Realized (since 1926)| 0.36    |           |                        |
| Realized (since 1948)| -0.42   |           |                        |
| Realized (since 1990)| -0.61   |           |                        |

**Note:** Panel A reports the skewness of changes of employment, capacity utilization, industrial production (beginning both in 1948 and in 1960), GDP, consumption and investments. The first column reports the skewness of monthly changes, the second column the skewness of quarterly changes. Panel B reports the realized skewness of S&P 500 monthly returns in different periods, as well as the implied skewness computed by the CBOE using option prices.
A.1 Construction of model-free implied volatility, $EV_n$

In this section we describe the details of the procedure we use to construct model-free implied volatility (market expectations of future volatility) at different horizons, starting from our dataset of end-of-day prices for American options on S&P 500 futures from the CME.

A.1.1 Main steps of construction of $EV$

A first step in constructing the model-free implied volatility is to obtain implied volatilities corresponding to the observed option prices. We do so using a binomial model.\(^1\) For the most recent years, CME itself provides the implied volatility together with the option price. For this part of the sample, the IV we estimate with the binomial model and the CME’s IV have a correlation of 99%, which provides an external validation on our implementation of the binomial model.

Once we have estimated these implied volatilities, we could in theory simply invert them to yield implied prices of European options on forwards. These can then be used to compute $EV$ directly as described in equation (15).

In practice, however, an extra step is required before inverting for the European option prices and integrating to obtain the model-free implied volatility. The model-free implied volatility defined in equation (15) depends on the integral of option prices over all strikes, but option prices are only observed at discrete strikes. We are therefore forced to interpolate option prices between available strikes and also extrapolate beyond the bounds of observed strikes.\(^2\) Following the literature, we fit a parametric model to the Black–Scholes implied volatilities of the options and use the model to then interpolate and extrapolate across all strikes (see, for example, Jiang and Tian (2007), Carr and Wu (2009), Taylor, Yadav, and Zhang (2010), and references therein). Only after this extra interpolation-extrapolation step, the fitted implied volatilities are then inverted to yield option prices and compute $EV$ according to equation (15). To interpolate and extrapolate the implied volatility curve, we use the SVI (stochastic volatility inspired) model of Gatheral and Jacquier (2014).

In the next sections, we describe in more detail the interpolation-extrapolation step of the procedure (SVI fitting) as well as our construction of $EV$ after fitting the SVI curve. Finally, we report a description of the data we use and some examples and diagnostics on the SVI fitting method.

A.1.2 SVI interpolation: theory

There are numerous methods for fitting implied volatilities across strikes. Homescu (2011) provides a thorough review. We obtained the most success using Gatheral’s SVI model (see Gatheral and Jacquier 2014). SVI is widely used in financial institutions because it is parsimonious but also known

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\(^1\)See for example Broadie and Detemple (1996) and Bakshi, Kapadia, and Madan (2003), among others.

\(^2\)See Jiang and Tian (2007) for a discussion of biases arising from the failure to interpolate and extrapolate.
The SVI model simply assumes a hyperbolic relationship between implied variance (the square of the Black–Scholes implied volatility) and the log moneyness of the option, \( k \) (log strike/forward price).

\[
\sigma^2_{BS}(k) = a + b \left[ \rho (k - m) + \sqrt{(k - m)^2 + \sigma^2} \right]
\]

where \( \sigma^2_{BS}(k) \) is the implied variance under the Black–Scholes model at log moneyness \( k \). SVI has five parameters: \( a, b, \rho, m, \) and \( \sigma \). The parameter \( \rho \) controls asymmetry in the variances across strikes. Because the behavior of options at high strikes has minimal impact on the calculation of model-free implied volatilities, and because we generally observe few strikes far above the spot, we set \( \rho = 0 \) (in simulations with calculating the VIX for the S&P 500 – for which we observe a wide range of options – we have found that including or excluding \( \rho \) has minimal impact on the result).

We fit the parameters of SVI by minimizing the sum of squared fitting errors for the observed implied volatilities. Because the fitted values are non-linear in the parameters, the optimization must be performed numerically. We follow the methodology in Zeliade (2009) to analytically concentrate \( a \) and \( b \) out of the optimization. We then only need to optimize numerically over \( \sigma \) and \( m \) (as mentioned above, we set \( \rho = 0 \)). We optimize with a grid search over \( \sigma \times m = [0.001, 10] \times [-1, 1] \) followed by the simplex algorithm.

For many date/firm/maturity triplets, we do not have a sufficient number of contract observations to fit the implied volatility curve (i.e. sometimes fewer than four). We therefore include strike/implied volatility data from the two neighboring maturities and dates in the estimation. The parameters of SVI are obtained by minimizing squared fitting errors. We reweight the observations from the neighboring dates and maturities so that they carry the same amount of weight as the observations from the date and maturity of interest. Adding data in this way encourages smoothness in the estimates over time and across maturities but it does not induce a systematic upward or downward bias. We drop all date/firm/maturity triplets for which we have fewer than four total options with \( k < 0 \) or fewer than two options at the actual date/firm/maturity (i.e. ignoring the data from the neighboring dates and maturities).

When we estimate the parameters of the SVI model, we impose conditions that guarantee the absence of arbitrage. In particular, we assume that \( b \leq \frac{4}{(1+|\rho|)T} \), which when we assume \( \rho = 0 \), simplifies to \( b \leq \frac{1}{T} \). We also assume that \( \sigma > 0.0001 \) in order to ensure that the estimation is well defined. Those conditions do not necessarily guarantee, though, that the integral determining the model-free implied volatility is convergent (the absence of arbitrage implies that a risk-neutral probability density exists – it does not guarantee that it has a finite variance). We therefore eliminate observations where the integral determining the model-free implied volatility fails to converge numerically. Specifically, we eliminate observations where the argument of the integral
does not approach zero as the log strike rises above two standard deviations from the spot or falls more than five standard deviations below the strike (measured based on the at-the-money implied volatility).

A.1.3 Construction of $EV$ from the SVI fitted curve

After fitting the SVI curve for each date and maturity, we compute the integral in equation (15) numerically, over a range of strikes from -5 to +2 standard deviations away from the spot price. We then have $EV$ for every firm/date/maturity observation. The model-free implied volatilities are then interpolated (but not extrapolated) to construct $EV$ at maturities from 1–6 months for each firm/date pair.

A.1.4 Data description and diagnostics of SVI fitting

Our dataset consists of 2.3 million end-of-day prices for all American options on S&P 500 futures from the CME.

When more than one option (e.g. a call and a put) is available at any strike, we compute IV at that strike as the average of the observed IVs. We keep only IVs greater than zero, at maturities higher than 9 days and lower than 2 years, for a total of 1.9 million IVs. The number of available options has increased over time, as demonstrated by Figure A.2 (top panel), which plots the number of options available for $EV$ estimation in each year.

The maturity structure of observed options has also expanded over time, with options being introduced at higher maturities and for more intermediate maturities. Figure A.1 (top panel) reports the cross-sectional distribution of available maturities in each year to estimate the term structure of the model-free implied volatility. The average maturity of available options over our sample was 4 months, and was relatively stable. The maximum maturity observed ranged from 9 to 24 months and varied substantially over time.

Crucial to compute the model-free implied volatility is the availability of IVs at low strikes, since options with low strikes receive a high weight in the construction of $EV$. The bottom panel of Figure A.1 reports the minimum observed strike year by year, in standard deviations below the spot price. In particular, for each day we computed the minimum available strike price, and the figure plots the average of these minimum strike price across all days in each year; this ensures that the number reported does not simply reflect outlier strikes that only appear for small parts of each year.

Figure A.1 shows that in the early part of our sample, we can typically observe options with strikes around 2 standard deviations below the spot price; this number increases to around 2.5 towards the end of the sample.

3In general this range of strikes is sufficient to calculate $EV$. However, the model-free implied volatility technically involves an integral over the entire positive real line. Our calculation is thus literally a calculation of Andersen and Bondarenko’s (2007) corridor implied volatility. We use this fact also when calculating realized volatility.
These figures show that while the number of options was significantly smaller at the beginning of the sample (1983), the maturities observed and the strikes observed did not change dramatically over time.

Figure A.3 shows an example of the SVI fitting procedure for a specific day in the early part of our sample (November 7th 1985). Each panel in the figure corresponds to a different maturity. On that day, we observe options at three different maturities, of approximately 1, 4, and 8 months. In each panel, the x’s represent observed IVs at different values of log moneyness $k$. The line is the fitted SVI curve, that shows both the interpolation and the extrapolation obtained from the model.

Figure A.4 repeats the exercise in the later part of our sample (Nov. 1st 2006), where many more maturities and strikes are available.

Both figures show that the SVI model fits the observed variances extremely well. The bottom panel of Figure A.2 shows the average relative pricing error for the SVI model in absolute value. The graph shows that the typical pricing error for most of the sample is around 0.02, meaning that the SVI deviates from the observed IV by around 2% on average. Only in the very first years (up to 1985) pricing errors are larger, but still only around 10% of the observed IV.

Overall, the evidence in this section shows that our observed option sample since 1983 has been relatively stable along the main dimensions that matter for our analysis – maturity structure, strikes observed, and goodness of fit of the SVI model.
Figure A.1: Maturities and strikes in the CME dataset

Note: Top panel reports the distribution of maturities of options used to compute the VIX in each year, in months. Bottom panel reports the average minimum strike in each year, in standard deviations below the forward price. The number is obtained by computing the minimum observed strike in each date and at each maturity (in standard deviations below the forward price), and then averaging it within each year to minimize the effect of outliers.
Figure A.2: Number of options to construct the VIX and pricing errors

**Note:** Top panel reports the number of options used to compute the VIX in each year, in thousands. Bottom panel reports the average absolute value of the pricing error of the SVI fitted line relative to the observed implied variances, in proportional terms (i.e. 0.02 means absolute value of the pricing error is 2% of the observed implied variance).
Figure A.3: SVI fit: 11/7/1985

Note: Fitted implied variance curve on 11/7/1987, for the three available maturities. X axis is the difference in log strike and log forward price. x’s correspond to the observed implied variances, and the line is the fitted SVI curve.
Figure A.4: SVI fit: 11/1/2006

Note: Fitted implied variance curve on 11/1/2006, for the three available maturities. X axis is the difference in log strike and log forward price. X’s correspond to the observed implied variances, and the line is the fitted SVI curve. On 11/1/2006 also a maturity of 5 months was available (not plotted for reasons of space).
Figure A.5: Impulse response functions from VAR (ordering \( ev \) first and \( rv \) second)

Note: The figure shows impulse response functions (with 90% and 95% CI) of \( rv \), volatility expectations \( ev \), employment and industrial production to shocks to \( ev \) and \( rv \), in a VAR with \( rv \), \( ev \), federal funds rate, log employment and log industrial production. Impulse response functions reported correspond to structural shocks with Choleski decomposition (in the order reported above). Sample covers 1983-2014. All macroeconomic series were detrended with a one-sided HP filter.
Note: The figure shows impulse response functions (with 90% and 95% CI) from a VAR with $rv$, volatility expectations $ev$, and the macroeconomic series from Basu and Bundick (2015): GDP (Y), consumption (C), investment (I), hours (H), the GDP deflator (DEF), M2 and the FFR. The figure reports IRF of Y, C, I, H to shocks to $rv$ and $ev$. Impulse response functions reported correspond to structural shocks with Choleski decomposition (in the order reported above). First row shows the response to a one-standard-deviation shock to $rv$. Second row shows the response to a one-standard-deviation shock to $ev$. Third row shows the difference in the response to the two shocks. Sample covers 1983-2014.
Figure A.7: Robustness (I): response of Employment to RV and expectations shocks across specifications

(a) Without detrending the macroeconomic time series

(b) VAR estimated at the quarterly frequency

(c) Subperiod 1988-2006 (excluding 1987 crash and financial crisis)

(d) Ordering RV and expectations last in the VAR

Note: The Figure reports the response of employment to RV shocks (left panels) and volatility expectations (middle panels) in different specification, with the difference in the right panel. Each row of the figure corresponds to a different model specification. Row (a) does not detrend the macroeconomic time series. Row (b) repeats the exercise using quarterly, rather than monthly, data. Row (c) estimates the VAR in the subsample 1988-2006, which excludes both RV peaks (1987 crash and financial crisis). Row (d) orders the RV shock second to last and the expectation shock last in the VAR.
Figure A.8: Robustness (II): response of Employment to RV and expectations shocks across specifications

(a) Using 6-month ATM IV

(b) Using 1-month EV

(c) Adding the S&P 500 level as first shock

(d) Using RV and EV in levels, not logs

**Note:** The Figure reports the response of employment to RV shocks (left panels) and volatility expectations (middle panels) in different specification, with the difference in the right panel. Each row of the figure corresponds to a different model specification. Row (a) uses the 6-month implied volatility from at-the-money options to measure expectations. Row (b) uses the 1-month EV rather than the 6-month EV. Row (c) adds as a first variable in the VAR (and in the Choleski decomposition) the level of the S&P 500, as in Bloom (2009). Row (d) uses RV and EV in levels, not logs.
Figure A.9: Robustness (III): response of IP to RV and expectations shocks across specifications

(a) Without detrending the macroeconomic time series

(b) VAR estimated at the quarterly frequency

(c) Subperiod 1988-2006 (excluding 1987 crash and financial crisis)

(d) Ordering RV and expectations last in the VAR

**Note:** The Figure reports the response of IP to RV shocks (left panels) and volatility expectations (middle panels) in different specification, with the difference in the right panel. Each row of the figure corresponds to a different model specification. Row (a) does not detrend the macroeconomic time series, Row (b) repeats the exercise using quarterly, rather than monthly, data. Row (c) estimates the VAR in the subsample 1988-2006, which excludes both RV peaks (1987 crash and financial crisis). Row (d) orders the RV shock second to last and the expectation shock last in the VAR.
Figure A.10: Robustness (IV): response of IP to RV and expectations shocks across specifications

(a) Using 6-month ATM IV

(b) Using 1-month EV

(c) Adding the S&P 500 level as first shock

(d) Using RV andEV in levels, not logs

**Note:** The Figure reports the response of IP to RV shocks (left panels) and volatility expectations (middle panels) in different specification, with the difference in the right panel. Each row of the figure corresponds to a different model specification. Row (a) uses the 6-month implied volatility from at-the-money options to measure expectations. Row (b) uses the 1-month EV rather than the 6-month EV. Row (c) adds as a first variable in the VAR (and in the Choleski decomposition) the level of the S&P 500, as in Bloom (2009). Row (d) uses RV and EV in levels, not logs.

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