

THE IMPACT OF INDUSTRIALIZED COUNTRIES' MONETARY POLICY ON EMERGING ECONOMIES

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VERY PRELIMINARY AND INCOMPLETE

Abstract

In this paper I study how the low interest rate policies adopted by industrialized countries after the 2008 financial crisis may have impacted the economic performance of emerging countries. Although these policies may have reduced outflows of capital away from emerging countries, the economic performance of these countries has deteriorated more than in industrial countries. I propose a model where, contrary to the more conventional view, lower interest rates in industrialized countries could have negative macroeconomic consequences for emerging countries.

Introduction

Following the 2008 financial crisis, many countries in the industrialized world have pursued expansionary monetary policies that resulted in lower interest rates. Figure 1 plots the policy rates for the major industrialized countries and shows that, with the exception of Japan, they have all lowered the interest rates after 2008. Japan is an exception because the policy rate was already close to zero before the financial crisis.

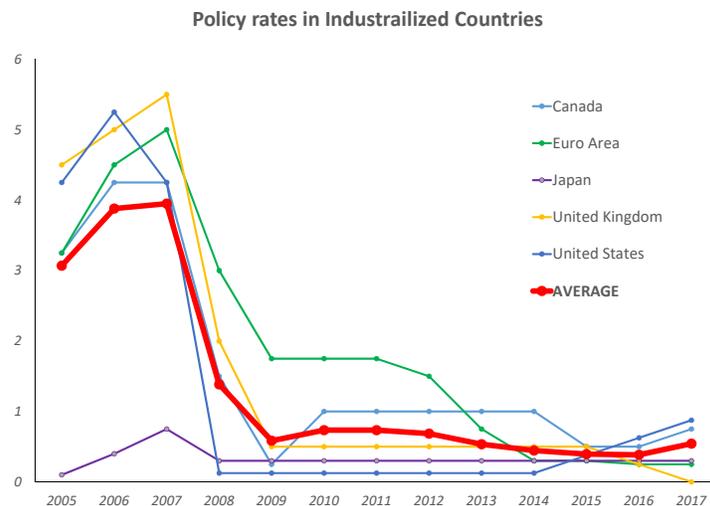


Figure 1: Policy interest rates in Industrialized Countries, 2005-2017. Sources: *International Financial Statistics, IMF*. The graph plots ‘Policy-Related Interest Rates’. If unavailable it plots the ‘Discount Rate’. The ‘AVERAGE’ is the un-weighted mean.

During the same period we observe a change in capital flows between industrialized and emerging countries as indicated by the current account (left panel of Figure 2). While emerging countries were net exporters of capital before the crisis (that is, they had positive current account balances), the post-crisis period shows a re-balancing of the current account. This is also noticeable by looking at the more liquid components of the financial account: the net flows of portfolio debt and international reserves (see right panel of Figure 2).¹

The fact that the capital flows re-balancing arose in conjunction with the lower interest rates in industrialized countries is consistent—although it is not a proof of it—with the view that loose monetary policies in industrialized countries increased the search for higher yields in emerging countries. The goal of this paper is to study the macroeconomic consequences of these policies and the associated capital re-balancing for emerging countries.

The conventional view is that higher inflows of capital (or lower outflows)

¹A recent study by the International Monetary Funds, IMF (2016), shows that the flows of capital to emerging countries has slowed down after the crisis. This study, however, uses only private flows which exclude foreign reserves. In my study, instead, I focus on the overall net flows of capital to emerging countries which, abstracting from errors and omissions, corresponds to the Current Account Balance.

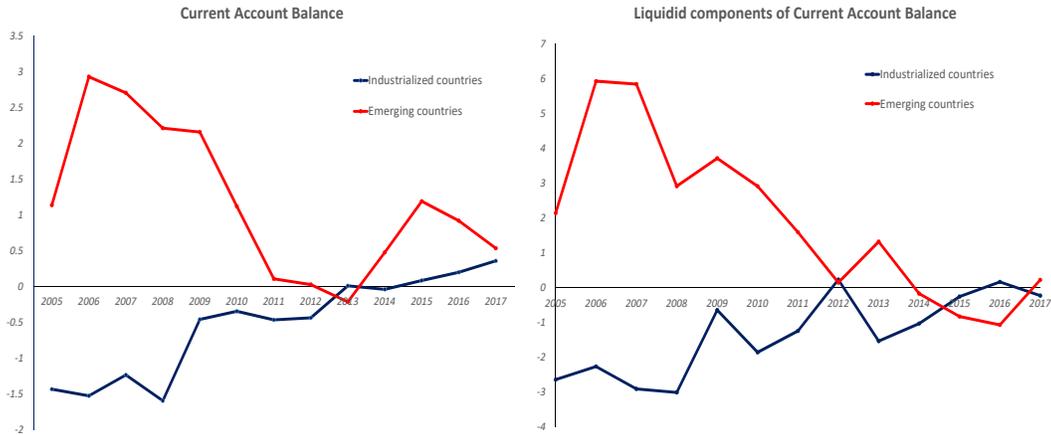


Figure 2: Net flows of capital as a percentage of GDP, 2005-2017. Sources: *Balance of Payments Analytic, IMF*. The liquid components of the current account shown in the second panel is the net flows of portfolio debt and foreign reserves. The aggregates are constructed using GDP at nominal exchange rates. **Emerging countries:** Argentina, Brazil, Bulgaria, Chile, China, Hong.Kong, Colombia, Estonia, Hungary, India, Indonesia, South Korea, Latvia, Lithuania, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Romania, Russia, South Africa, Thailand, Turkey, Ukraine, Venezuela. **Industrialized countries:** Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United.Kingdom, United.States.

in emerging countries would lower the local interest rate and expand domestic credit, which in turn create the conditions for a macroeconomic boom. In the long-run it may also increase fragility since certain sectors of the economy become more leveraged. But, at least initially, it should stimulate growth in emerging countries. This, however, is not what happened to emerging countries after the financial crisis.

Figure 3 shows that GDP growth in emerging countries slowed down substantially after the financial crisis, while in industrialized countries the overall growth did not change much. As a result, the growth differential between emerging and industrialized countries dropped significantly after the crisis. This is somewhat surprising because most of these countries did not experience the financial turbulence experienced by industrialized countries, at least not to that extent. Nevertheless, the real sector of the economy

did contract during the crisis, which is not surprising given the high degree of global integration in real and financial markets. However, the fact that the post-crisis growth fell more than in industrialized countries is somewhat surprising and suggests that the capital flows re-balancing has not been very helpful for the macro-economy of these countries. Again, this is at odd with the conventional view that lower interest rates and inflows of capital bring macroeconomic benefits to the receiving countries.²

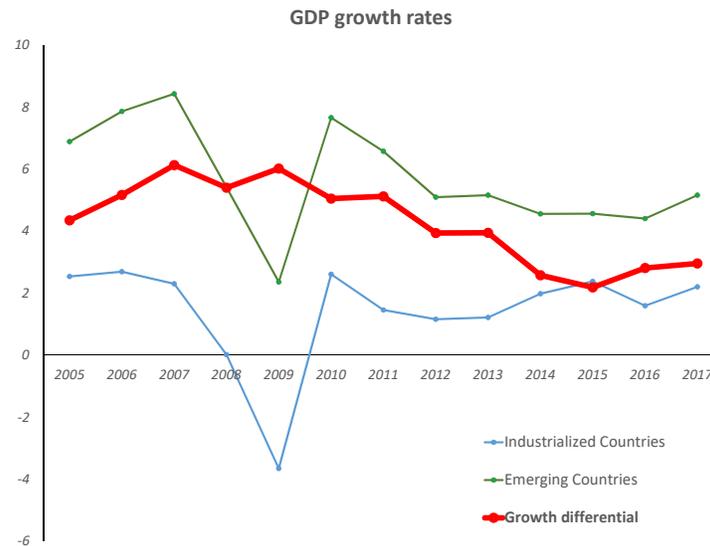


Figure 3: Growth rate of GDP in Industrialized and Emerging Countries, 2005-2017. Sources: *World Development Indicators*, *World Bank*. The aggregates are constructed by weighting country real growth rate in GDP by GDP at nominal exchange rates. **Emerging countries:** Argentina, Brazil, Bulgaria, Chile, China, Hong.Kong, Colombia, Estonia, Hungary, India, Indonesia, South Korea, Latvia, Lithuania, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Romania, Russia, South Africa, Thailand, Turkey, Ukraine, Venezuela. **Industrialized countries:** Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United.Kingdom, United.States.

²An alternative interpretation of why the outflows of capital from emerging countries fell down is because the macroeconomic slowdown of industrialized countries made investments in these countries less attractive. This interpretation, however, is inconsistent with the fact that the growth rate of emerging countries slowed down even more than in industrialized countries. Therefore, in terms of growth prospects, industrial countries remained ‘relatively’ more attractive than emerging countries.

One limitation of the conventional view is that it captures only one of the possible mechanisms through which interest rates affect the macro-economy. In reality, there could be other mechanisms besides the lowest cost of investment. In particular, it ignores the fact that interest rates also affect savings. When the interest rate drops, savers have less incentive to save and, as a result, they hold less financial assets. To the extent that the holding of financial assets affects real economic decisions, including investments, this may have a negative macroeconomic effect. It not only creates the conditions for greater fragility (due to higher leverage) but it also discourages savings which could impair long term growth.

I show this result with a model economy calibrated to emerging countries. There are two production sectors in the economy. The first sector produces output with a risky technology that uses labor as the only input of production. Risk derives from the fact that production is carried out by individual entrepreneurs and the production function is subject to an ‘uninsurable’ idiosyncratic shock. The idiosyncratic shock leads producers to save for precautionary reasons and when they hold more financial wealth, they are willing to take more risk by increasing the scale of production. The second sector, instead, produces output with a non-reproducible asset but with lower incidence of ‘uninsurable’ idiosyncratic shocks. Because of the lower (uninsurable) risk, producers in the second sector save less and, in equilibrium, they become net borrowers.

I think of the first sector as the growth-enhancing sector while the second as the sector that produces services from less flexible inputs. An example is housing. Arguably, growth enhanced activities tend to be individually riskier than activities for which a higher component of income derives from rents. I refer to the first sector as ‘growth-sector’ and the second as ‘rent-sector’.

Within the model, the impact of an expansionary monetary policy in industrialized countries is captured by a reduction in the world interest rate. This has two consequences for emerging economies. First, the rent-sector borrows more because the cost of borrowing declines. This increases the demand for non-reproducible assets which in turn raises its market value. Therefore, a consequence of the lower interest rate is an asset price boom in the rent-sector of the economy. However, since the asset used in production is not reproducible, production does not change in this sector. Intuitively, cheaper credit increases the demand for houses but the services from existing houses remain the same. If we assume that new houses can be produced, this would stimulate new constructions but slowly.

The impact of a lower interest rate in the growth-sector is different. The lower interest rate reduces savings which in turn generates a decline in growth enhancing production. Therefore, even though the change in external monetary policy generates an asset price boom in certain sectors (like real estates), it could decrease the overall economic growth of emerging countries.

In addition to lower growth, the higher leverage in the rent-sector increases future macroeconomic instability. Future internal or external shocks could create the conditions for larger re-adjustments when the economy is more leveraged, which in turn creates larger macroeconomic contractions. A monetary policy reversal in industrialized countries may be one of the external forces that could induce a financial re-adjustment in emerging countries. The policy reversal would cause a price drop for non-reproducible assets, which could trigger default in the rent-sector. This, effectively, redistributes wealth away from savers (in the growth sector) to borrowers (in the rent-sector). The capital losses experienced by savers in the growth-sector would then trigger a decline in real growth.

The paper is organized as follows. Section 1 develops the theoretical model. Section 2 uses the calibrated model to evaluate the impact of industrial countries' monetary policy on emerging countries. Section 3 extends the model by allowing for endogenous growth. Section 5 summarizes the results and concludes.

1 Model

I consider a small open economy model representative of emerging countries. Modeling emerging countries as a small opening economy is obviously a limitation since, as a group, these countries are not small relative to the world economy. However, by limiting the analysis to a small economy I can treat the world interest rate as exogenous, which simplifies considerably the characterization of the model.

The economy has two sectors: the entrepreneurial sector and the household sector. I start with the description of the entrepreneurial sector.

1.1 Entrepreneurial sector

In the entrepreneurial sector there is a unit mass of atomistic entrepreneurs, indexed by i , with lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t^i),$$

where c_t^i is the consumption of entrepreneur i at time t .

Entrepreneurs are business owners producing a single good with the production technology described below. Consumption should be thought as dividends paid by the firm and the concavity of the utility function captures the risk aversion of entrepreneurs or managers in the case of separation between ownership and management.

Each firm operates the production technology

$$y_t^i = z_t^i h_t^i,$$

where h_t^i is the input of labor supplied by households (as described below) at the market wage w_t , and $z_t^i = A_t \pi_t^i$ is productivity.

Productivity is the product of two components. The first component, A_t , is the ‘aggregate’ country-specific productivity and the second, π_t^i , is an ‘idiosyncratic’ shock. The aggregate productivity A_t is common for all entrepreneurs and the evolution over time will be described later. The idiosyncratic component, $\pi_t^i \in [\underline{\pi}, \bar{\pi}]$, is distributed independently and identically across entrepreneurs and time with probability distribution $\Gamma(\pi)$.

As in Arellano, Bai, and Kehoe (2011), the input of labor h_t^i is chosen before observing the idiosyncratic component of productivity, π_t^i . This implies that the choice of labor is risky. To insure consumption smoothing, entrepreneurs have access to two types of bonds: domestic bonds, denoted by b_t^i , and foreign bonds, denoted by f_t^i . Domestic bonds are liabilities issued by households at price q_t^b while foreign bonds are liabilities issued by industrialized countries at price q_t^f .

There are two differences between domestic and foreign bonds. First, while the issuers of domestic bonds (households) could default on their liabilities, foreign bonds are always repaid. I will relax this assumption later in the paper. Second, while the supply of foreign bonds is perfectly elastic and the price q_t^f is exogenous (given that this is a small open economy), the price of domestic bonds q_t^b is endogenous and will reflect the probability of

default as well as the price of foreign bonds. Since bonds, domestic or foreign, cannot be contingent on the realization of productivity, they provide only partial insurance.

An entrepreneur i enters period t with domestic and foreign bonds, b_t^i and f_t^i . In the event of a domestic financial crisis, the entrepreneur incurs financial losses that are proportional to the holding of domestic bonds. Denoting by δ_t the unit loss realized at the beginning of the period on domestic bonds, the residual value of the domestic bonds are $\tilde{b}_t^i = (1 - \delta_t)b_t^i$ while the value of foreign issued bonds remains f_t^i . The unit loss δ_t is an endogenous stochastic variables and will be determined in equilibrium.³

Given the residual wealth $\tilde{b}_t^i + f_t^i$, the entrepreneur chooses the input of labor h_t . Then, after the observation of the idiosyncratic productivity and, therefore, z_t^i , the entrepreneur chooses consumption c_t^i and purchases of new domestic bonds b_{t+1}^i at prices q_t^b , and the new foreign bonds f_{t+1}^i at price q_t^f . The budget constraint, after the observation of productivity is

$$c_t^i + q_t^b b_{t+1}^i + q_t^f f_{t+1}^i = \tilde{b}_t^i + f_t^i + (z_t^i - w_t)h_t^i. \quad (1)$$

Because labor h_t^i is chosen before the observation of z_t^i , while the saving decision is made after observing z_t^i , it will be convenient to define the entrepreneur's wealth after production

$$a_t^i = \tilde{b}_t^i + f_t^i + (z_t^i - w_t)h_t^i.$$

Given the timing assumption, the input of labor h_t^i depends on $\tilde{b}_t^i + f_t^i$ while the portfolio decisions b_{t+1}^i and f_{t+1}^i depend on a_t^i . To further clarify the timing, it would be convenient to summarize the sequence of events in each period as taking place in three sequential stages:

1. **Stage 1:** The entrepreneur enters the period with financial assets b_t^i and f_t^i , and observes the aggregate variable δ_t . The realization of financial losses on domestic bonds brings the residual value to $\tilde{b}_t^i = (1 - \delta_t)b_t^i$.
2. **Stage 2:** Given \tilde{b}_t^i and f_t^i , the entrepreneur chooses the input of labor h_t^i before knowing the idiosyncratic productivity π_t^i . Market clearing in the labor market determines the wage rate w_t .

³For the moment I abstract from the possibility that a crisis could also arise in industrialized countries and focus only on the implications of interest rate policy chosen by industrialized countries.

3. **Stage 3:** Productivity $z_t^i = A_t \pi_t^i$ becomes known. The end-of-period wealth $a_t^i = \tilde{b}_t^i + f_t^i + (z_t^i - w_t)h_t^i$ is in part used for consumption, c_t^i , and in part (saved) to purchase new domestic and foreign bonds, $q_t^b b_{t+1}^i + q_t^f f_{t+1}^i$.

The next step is to characterize the entrepreneur's policies before and after observing the idiosyncratic productivity.

Lemma 1.1 *The optimal entrepreneur's policies are*

$$\begin{aligned} h_t^i &= \phi_t(\tilde{b}_t^i + f_t^i), \\ c_t^i &= (1 - \beta)a_t^i, \\ q_t^b b_{t+1}^i &= \beta \theta_t a_t^i, \\ q_t^f f_{t+1}^i &= \beta(1 - \theta_t)a_t^i. \end{aligned}$$

where ϕ_t and θ_t satisfy

$$\begin{aligned} \mathbb{E}_{z_t^i} \left\{ \frac{z_t^i - w_t}{1 + (z_t^i - w_t)\phi_t} \right\} &= 0 \\ \mathbb{E} \left\{ \frac{q_t^b}{(1 - \delta_{t+1})q_t^f \theta_t + q_t^b(1 - \theta_t)} \right\} &= 1 \end{aligned}$$

Proof 1.1 *See Appendix A.*

The demand for labor, which is chosen before observing the realization of idiosyncratic productivity, is linear in financial wealth $\tilde{b}_t^i + f_t^i$. The proportional factor ϕ_t is defined by the condition $\mathbb{E}_{z_t^i} \left\{ \frac{z_t^i - w_t}{1 + (z_t^i - w_t)\phi_t} \right\} = 0$. This is derived from the first order condition of labor and it is the same for all entrepreneurs.

The factor ϕ_t captures the importance of risk aversion for determining the demand for labor. Because productivity is unknown when an entrepreneur chooses the scale of production, labor is risky and entrepreneurs require a positive profit over the cost of labor as a premium in compensation for the risk. As a result, the expected marginal product of labor is higher than the wage rate, that is, $\mathbb{E}_t z_t^i > w_t$. Furthermore, higher is the expected unit profit, $\mathbb{E}_t z_t^i - w_t$, and higher is the scale of production ϕ_t . On the other hand, if

we fix the expected unit profit, the scale of production decreases with the volatility of productivity (risk).

Since the distribution of z_t^i does not change over time, the only ‘endogenous’ variable that affects ϕ_t is the wage rate w_t . I make this dependence explicit by using the function $\phi_t(w_t)$, which is strictly decreasing in w_t .

Lemma 1.1 also indicates that entrepreneurs allocate their end-of-period wealth between consumption and savings according to the fixed factor β . This is a property that derives from the log specification of the utility function. Finally, a fraction θ_t of savings are allocated to domestic bonds and the remaining fraction $1 - \theta_t$ to foreign bonds. This is determined by the first order condition for the optimal choice of f_t that takes the form reported in the lemma. Since this condition depends only on aggregate variables, θ_t is the same for all entrepreneurs, which explains the omission of subscript i .

The aggregate demand for labor is derived by aggregating individual demands and can be written as

$$H_t = \phi_t(w_t) \int_i (\tilde{b}_t^i + f_t^i) = \phi_t(w_t) \tilde{B}_t + F_t,$$

where capital letters denote aggregate variables (upon aggregation over all entrepreneurs).

The aggregate demand for labor depends negatively on the wage rate and positively on the aggregate financial wealth of entrepreneurs. The dependence of wealth does not derive from entrepreneurs being financially constrained. In fact, labor does not need to be financed since it is not paid in advance. Instead, this property derives from the fact that employment is risky and entrepreneurs are willing to hire more workers only if they hold a higher wealth buffer.

1.2 Household sector

There is a unit mass of atomistic households with utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (c_t - A_t h_t),$$

where c_t is consumption and h_t is employment. Households are homogeneous and they do not face idiosyncratic shocks.

The assumption that households have linear utility in consumption simplifies the characterization of the equilibrium and allows for some analytical results, without affecting the key properties of the model. As long as households do not face idiosyncratic risks (or the idiosyncratic risk is significantly lower than the risk faced by entrepreneurs), the model would display similar properties even if households were risk averse.

The linear specification of the dis-utility from working can be justified with the indivisibility of labor, which is a common assumption in many business cycle models. The dependence of the dis-utility from aggregate productivity A_t guarantees balanced growth.

Households hold a non-reproducible asset which is available in fixed supply \bar{K} . Each unit of the asset produces A_t units of consumption goods to households but not to entrepreneurs. The productivity of the asset increases with the country-specific productivity, another assumption necessary to have balanced growth. The asset is divisible and can be traded by households at the market price p_t . I will interpret the fixed asset as residential houses and its production as housing services.⁴

Debt and default. Households can borrow l_t/R_{t-1} at the end of period $t - 1$ (R_{t-1} is the gross interest rate) with the promise to repay l_t in period t . At the beginning of period t , however, when the repayment l_t is due, the household could default on the debt.

In the event of default, creditors have the right to liquidate k_t and sell it at the liquidation price \tilde{p}_t . The liquidation price \tilde{p}_t at the ‘beginning of the period’ could differ from the price p_t at the ‘end of the period’ when houses are traded. In particular, I assume that with some probability λ the liquidation price drops to ξA_{t-1} . The parameter ξ is sufficiently small so that the liquidation price at the beginning of the period drops below the price at the end of the period, that is, $\xi A_{t-1} < p_t$.

Let ε_t be a random variable that takes the value of 0 with probability λ and 1 with probability $1 - \lambda$. The liquidation price takes the form

$$\tilde{p}_t = \begin{cases} \xi A_{t-1}, & \text{if } \varepsilon_t = 0 \\ p_t, & \text{if } \varepsilon_t = 1 \end{cases}. \quad (2)$$

⁴In principle, I could allow entrepreneurs to hold and trade houses. However, if houses provide services only to households and renting them involves substantial agency problems, in equilibrium entrepreneurs would choose not to hold them.

The mechanism leading to the price drop is described in details in Appendix C and it is based on self-fulfilling expectations. In that context, the variable ε_t is a sunspot shock and λ represents the probability that the sunspot shock takes a value that triggers negative self-fulfilling expectations.

Once \tilde{p}_t becomes known at the beginning of period t , households could use the threat of default to renegotiate the outstanding liabilities l_t . Of course, the debt will be renegotiated only if the liabilities are bigger than the liquidation value, that is, $l_t > \tilde{p}_t k_t$. Under the assumption that households have the whole bargaining power, the debt will be renegotiated to the liquidation value. Thus, the post-renegotiation debt is

$$\tilde{l}(l_t, \tilde{p}_t k_t) = \begin{cases} l_t, & \text{if } l_t \leq \tilde{p}_t k_t \\ \tilde{p}_t k_t, & \text{if } l_t > \tilde{p}_t k_t \end{cases}. \quad (3)$$

I assume that renegotiation brings a cost that is increasing and convex in the size of the renegotiation $l_t - \tilde{p}_t k_t$, that is,

$$\varphi\left(\frac{l_t}{\tilde{p}_t k_t}\right) l_t = \begin{cases} 0, & \text{if } l_t \leq \tilde{p}_t k_t \\ \chi \left(\frac{l_t - \tilde{p}_t k_t}{l_t}\right)^2 l_t, & \text{if } l_t > \tilde{p}_t k_t \end{cases}. \quad (4)$$

Obviously, the cost is zero if there is no renegotiation, that is, the liabilities are smaller than the value of the house, $l_t \leq \tilde{p}_t k_t$. It becomes positive if the borrower renegotiates the debt, that is, $l_t > \tilde{p}_t k_t$. Besides the renegotiation cost, there are no penalties for the borrowers who will be able to re-enter the credit market immediately at the end of the period when the regular market for houses takes place (fresh-start).

The assumption of an immediate fresh-start is a simplification that makes the model tractable. Under this assumption, the household's budget constraint after renegotiation is

$$\tilde{l}(l_t, \tilde{p}_t k_t) + \varphi\left(\frac{l_t}{\tilde{p}_t k_t}\right) l_t + (k_{t+1} - k_t) p_t + c_t = \frac{l_{t+1}}{R_t} + w_t h_t + A_t k_t.$$

The gross interest rate R_t is not taken as given by the household but it depends on the borrowing decision. If the household borrows more, relatively to the value of the house, the expected repayment rate could be lower in the next period. This will be reflected in a higher interest rate on the loan.

Denote by \bar{R}_t the expected gross return from holding the debt issued in period t by ‘all’ households and repaid in period $t + 1$. This is the market return which is taken as given in a single borrowing transaction. Since households are atomistic and financial markets are competitive, the expected return on the debt issued by an ‘individual’ household must be equal to the aggregate expected return \bar{R}_t . Thus, the interest rate on the debt issued by an individual household must satisfy

$$\frac{l_{t+1}}{R_t} = \frac{\mathbb{E}_t \tilde{l}(l_{t+1}, \tilde{p}_{t+1} k_{t+1})}{\bar{R}_t}. \quad (5)$$

The left-hand-side is the amount borrowed in period t while the right-hand-side is the expected repayment in period $t + 1$, discounted by the market return \bar{R}_t . Since the household renegotiates in the next period if $l_{t+1} > \tilde{p}_{t+1} k_{t+1}$, the actual repayment could be lower than the original debt. Competition in financial intermediation requires that the left-hand-side of (5) is equal to the right-hand-side.

Equation (5) determines the interest rate R_t for an individual household. In equilibrium, of course, all households will make the same decisions and they all borrow at the same rate. However, in order to characterize the optimal decision of an individual household, we need to allow the household to deviate from other households, which in turn implies a deviation of the individual borrowing rate as determined by equation (5).

First order conditions. As for entrepreneurs, households’ decisions are made in three stages. In the first stage households decide whether to default on the debt. In the second stage, before the realization of aggregate productivity, they decide the supply of labor. In the third stage households choose investment in housing and the debt. Appendix B describes the households’ problem and derives the following first order conditions

$$w_t = A_t, \quad (6)$$

$$\frac{1}{R_t} = \beta + \Phi_t \left(\frac{l_{t+1}}{k_{t+1}} \right), \quad (7)$$

$$p_t = \beta \mathbb{E}_t \left(A_{t+1} + p_{t+1} \right) + \Psi_t \left(\frac{l_{t+1}}{k_{t+1}} \right). \quad (8)$$

The functions $\Phi_t(\cdot)$ and $\Psi_t(\cdot)$, derived in the appendix, are increasing in the ratio l_{t+1}/k_{t+1} . I refer to this ratio as leverage. Thus, according to

equation (??), when the expected return on household debt declines, leverage increases. According to equation (8) this implies that an increase in leverage raises the return from house ownership which in turn increases its price. Thus, a decline in the (expected) interest rate will be associated with higher leverage and higher price of houses.

1.3 General equilibrium

I will use capital letters to denote aggregate variables. The state variables at the beginning of the period are aggregate productivity, A_t , domestic and foreign bonds held by entrepreneurs, B_t and F_t , liabilities issued by households, L_t , and exogenous shock ε_t . To use a compact notation I denote the vector of state variables by $\mathbf{s}_t \equiv (A_t, B_t, F_t, L_t, \varepsilon_t)$.

The equilibrium is determined sequentially in three stages:

1. **Stage 1:** Given the shock ε_t , the liquidation price is ξA_t if $L_t \geq \xi A_t \bar{K}$ and $\varepsilon_t = 0$. Given the liquidation price, households choose whether to default. The renegotiated liabilities are

$$\tilde{L}_t = \begin{cases} \xi A_t \bar{K}, & \text{if } L_t \geq \xi A_t \bar{K} \text{ and } \varepsilon_t = 0 \\ L_t, & \text{otherwise} \end{cases}.$$

The post-renegotiation value of domestic bonds is $\tilde{B}_t = \tilde{L}_t$.

2. **Stage 2:** Given the post-renegotiation wealth $\tilde{B}_t + F_t$, entrepreneurs choose the demand for labor and households choose the supply. At this stage the idiosyncratic productivity π_t^i is unknown.

The aggregate demand for labor is $H_t^D = \phi_t(w_t)(\tilde{B}_t + F_t)$, which depends negatively on the wage rate w_t and positively on the aggregate wealth of entrepreneurs, $\tilde{B}_t + F_t$. The supply of labor is derived from the households' first order condition (6). Market clearing will then determine the wage rate w_t and employment H_t .

3. **Stage 3:** Idiosyncratic productivity π_t^i is realized. The wealth of entrepreneurs becomes $\tilde{B}_t + F_t + (A_t - w_t)H_t$, which is in part consumed and in part saved in new bonds, $q_t^b B_{t+1}$ and $q_t^f F_{t+1}$. Households choose the new loans, L_{t+1} , and houses, $K_{t+1} = \bar{K}$.

Market clearing in financial assets gives rise to the condition $B_{t+1} = L_{t+1}$. The net foreign asset position of the country is $q_t^f F_t$. Competition implies that the price paid by entrepreneurs to buy households' debt is consistent with the interest rate charged to households, that is, $q_t^b = 1/R_t$. Since $\bar{R}_t = R_t(1 - \mathbb{E}_{t+1}\delta_{t+1})$, we also have $q_t^b = (1 - \mathbb{E}_{t+1}\delta_{t+1})/\bar{R}_t$. As shown in Lemma 1.1, the optimal savings of entrepreneurs takes the form $q_t^b b_{t+1}^i + q_t^f f_{t+1}^i = \beta a_t^i$, where a_t^i is the end-of-period wealth. Aggregating over all entrepreneurs we obtain aggregate savings which are allocated to domestic and foreign bonds, that is,

$$q_t^b B_{t+1} + q_t^f F_{t+1} = \beta \int_i a_t^i. \quad (9)$$

The demand for domestic bonds is determined by the fraction θ_t of savings allocated to these bonds, that is,

$$q_t^b B_{t+1} = \theta_t \beta \int_i a_t^i \quad (10)$$

The supply of domestic bonds, instead, is derived from the borrowing decisions of households. From the first order condition (??) we have

$$\frac{1}{\bar{R}_t} = \beta \left[1 + \Phi_t \left(\frac{L_{t+1}}{\xi A_{t+1} \bar{K}} \right) \right].$$

Since in equilibrium $\bar{R}_t = R_t(1 - \mathbb{E}\delta_t)$ and $q_t^b = 1/R_t$, the first order condition can be rewritten as

$$q_t^b = \beta (1 - \mathbb{E}\delta_t) \left[1 + \Phi_t \left(\frac{L_{t+1}}{\xi A_{t+1} \bar{K}} \right) \right]. \quad (11)$$

Given the end-of-period wealth held by entrepreneurs, $\int_i a_t^i$, and aggregate productivity A_t , we can solve for θ_t , q_t^b , B_t , F_t , L_t using the market clearing condition in domestic bonds, $B_t = L_t$, equations (10) and (11), and entrepreneurs' first order conditions for the choice of domestic and foreign bonds,

$$\mathbb{E} \left\{ \frac{(1 - \delta_{t+1})q_t^f}{(1 - \delta_{t+1})q_t^f \theta_t + q_t^b (1 - \theta_t)} \right\} = 1$$

$$\mathbb{E} \left\{ \frac{q_t^b}{(1 - \delta_{t+1})q_t^f \theta_t + q_t^b (1 - \theta_t)} \right\} = 1$$

Proposition 1.1 *Suppose that A_t and q_t^f are both constant. Furthermore, suppose that bonds are always repaid (no default), that is, $\delta_t = 1$, $\tilde{B}_t = B_t$ and $\tilde{L}_t = L_t$. The economy converges to a steady state where $q^b = 1/R > \beta$ and households borrow from entrepreneurs.*

Proof 1.1 *See Appendix ??*

The reason entrepreneurs hold domestic bonds even if their return is lower than the intertemporal discount rate is because they face uninsurable risks and bonds provide consumption insurance. When households can default and the aggregate productivity is stochastic, the economy may not reach a steady state but displays stochastic dynamics in response to fluctuations in the liquidation price.

2 Quantitative analysis

The model is calibrated annually using data for the period 1991-2005. Starting in 2005, I simulate the model until 2017. The list of industrialized and emerging countries is provided in Figure 2.

2.1 Calibration

The discount factor is set to $\beta = 0.93$, implying an annual intertemporal discount rate of about 7%.

Total production is the sum of entrepreneurial output, $A_t H_t$, and housing services, $A_t \bar{K}$. Thus, aggregate output is $Y_t = A_t(H_t + \bar{K})$. Because in the model there is no capital accumulation, the empirical counterpart of aggregate output is Gross Domestic Product minus Investment. I start with the assumption that A_t is constant and normalized to 1. This should be interpreted as the de-trended value of aggregate productivity for the group of emerging countries during the period 1991-2005.

To pin down the value of \bar{K} I use the share of housing services in net GDP (net of investment), which in the model is equal to $\bar{K}/(H_t + \bar{K})$. Unfortunately, data for the share of housing services is not available for many countries. To obviate this problem, I impose that emerging countries have the same share of housing services in output (GDP minus investment in the data) and use the US share as the calibration target. Based on NIPA data,

the average share of housing services in net GDP over the period 1991-2013 is 12.2%. Thus, I calibrate \overline{K} using the condition

$$\frac{\overline{K}}{\overline{H} + \overline{K}} = 0.122,$$

where \overline{H} is the average employment-to-population ratio over the period 1991-2005 for emerging countries. Using data from World Development Indicators (WDI) I set $\overline{H} = 0.449$.

The probability that the liquidation price drops to ξA_t , which I interpret as a crisis, is set to $\lambda = 0.02$. Thus, crises are very low probability events. On average, once every fifty years. Similar numbers have been used in the literature. See for example Bianchi and Mendoza (2013).

The stochastic process for the uninsurable idiosyncratic productivity π follows a truncated normal distribution with zero mean and standard deviation σ_π . The standard deviation σ_π determines the ‘demand’ of assets (in the spirit of Mendoza, Quadrini, and Ríos-Rull (2009)). Higher values of σ_π increase the demand for domestic and foreign bonds. I set the standard deviation of the idiosyncratic shock to 0.1 which can be justified by firm-level empirical volatility. The parameter ξ , instead, determines the recovery value of loans when the housing market drops and there is default. This in turns determines the incentive of households to borrow and, therefore, the ability of the country to create financial assets (in the spirit of Caballero, Farhi, and Gourinchas (2008)). Another parameter that affects the creation of financial assets is the cost of renegotiation captured by the parameter χ . Unfortunately, I do not have direct information to calibrate this parameter and I set it to $\chi = 5$. Then, to calibrate ξ I use the ratio of domestic credit to net GDP, which in the model corresponds to L_{t+1}/Y_t . For the group of emerging countries during the period 1991 to 2005 this ratio is equal to 49.6. However, some of these liabilities are held by other households. As a compromise I use half of this value as a calibration target for the model.

Finally the price of foreign bonds is set to $q_t^f = 0.96$ which corresponds to an interest rate of about 4 percent.

2.2 Quantitative results

I simulate the model for 113 years using a random sequence of draws of ε_t (sunspot shock). With probability $\lambda = 0.02$ the random draw is $\varepsilon_t = 0$ and

the liquidation price is $\tilde{p}_t = \xi$; with probability $1 - \lambda = 0.98$ the random draw is $\varepsilon_t = 1$ and the liquidation price is $\tilde{p}_t = p_t$. The first 100 years of simulation correspond to the pre-2005 period and the remaining 13 years correspond to the period from 2005 to 2017. The purpose of the pre-simulation of 100 periods is to eliminate the effect of initial conditions, that is, the values of the state variables.

Until 2008 the foreign interest rate is kept constant at the calibrated value of 4 percent. After 2008 the foreign interest rate drops to 1 percent.

In absence of sunspot shocks, the dynamics of the economy would be solely driven by changes in the interest rate. The presence of ε_t adds another source of fluctuations. The resulting simulation would then depend on the actual realization of these shocks. To better illustrate how these shocks affect the stochastic properties of the model, I repeat the simulation 1,000 times, with each simulation conducted over 100+13 years.

Simulation results Figure 4 plots the average as well as the 5th and 95th percentiles of the 1,000 repeated simulations. The range of variation between the 5th and 95th percentiles indicates the potential volatility at any point in time.

As a result of the interest rate drop in industrialized countries, households borrow more. Entrepreneurs, however, have less incentive to save and their wealth starts to decline. Since households borrow more while entrepreneurs save less, the country starts to export less capital. As can be seen in the third panel, the net foreign asset position of the country switches from positive to negative. Entrepreneurs are now borrowing from abroad. The lower interest rate has also a positive effect on housing prices since it lowers the financing cost of houses. The macroeconomic impact, however, is negative. As shown in the last panel of Figure 4, output declines on average. This is a direct consequence of the lower entrepreneurial savings shown in the third panel: as entrepreneurs hold less wealth, they choose a smaller scale of production in order to reduce the risk.

Figure 4 also shows the consequences of lower interest rates on macroeconomic volatility. As can be seen in the last panel, the distance between the 5th and 95th percentiles for output widens. This shows that lower interest rates not only reduce the level of economic activity but also increase volatility (higher fragility). This is because the economy becomes more leveraged and when a crisis arrives the wealth losses incurred by entrepreneurs

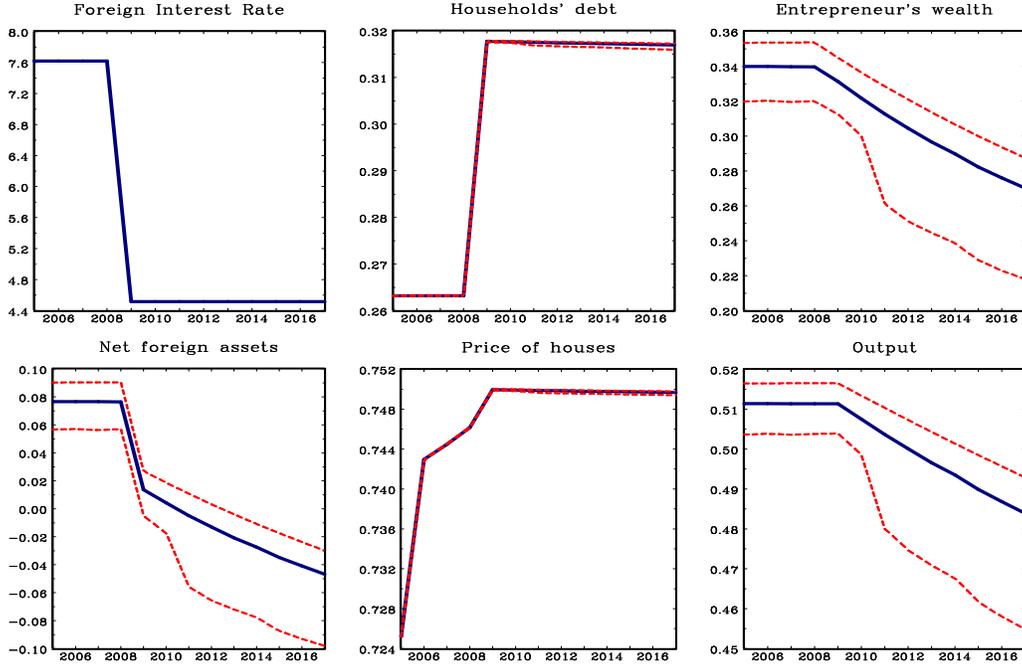


Figure 4: Response to low interest rates in Industrialized Countries, 2005-2017.

(due to default) are larger. Larger capital losses then have larger effects on production.

3 Endogenous growth

I now extend the model with endogenous growth. Following the literature, I introduce a production externality that depends on aggregate inputs of production. In the simpler version of the endogenous growth model, the productivity of an individual firm depends on the aggregate input of capital, that is,

$$y_t = A_t k_t^\alpha,$$

where $A_t = K_t^{1-\alpha}$. As the economy accumulates more capital, productivity increases and this leads to persistent growth.

In the model used in this paper, the production input is labor. Therefore, I assume that the externality is in the aggregate input of labor instead of capital. Furthermore, since labor is not reproducible, I assume that the

input of labor affects the ‘growth rate’ of aggregate productivity rather than its ‘level’, that is,

$$\frac{A_{t+1}}{A_t} = \kappa H_t.$$

One way to interpret this formulation is that there is learning by doing: higher labor (hours or employment) increases human capital which in turn affects productivity. Reinterpreting K_t as the stock of human capital, individual production takes the form

$$y_t = A_t h_t,$$

where $A_t = K_t$ and human capital evolves according to $K_{t+1} = \kappa K_t H_t$. The only difference with the more standard AK model is that the increase in (human) capital is not determined by savings but by time spent in the working place (learning-by-doing).

Simulation. Figure 5 plots the growth rate of output in response to the lower interest rate in industrialized countries. By making the growth rate of productivity endogenous, the model generates a slow down in growth as a result of the lower interest rates. The mechanism leading to the slow down works through the reduction in savings. Since financial assets have a lower return, entrepreneurs choose to hold less financial wealth. But when entrepreneurs hold less financial wealth they are less willing to take on production risk and reduce the scale of production. Through the externality, then, the lower production scale translates in lower growth. Although this does not prove that the slow down experienced by emerging countries was caused by the lower interest rates in industrialized countries, it is consistent with the theory proposed in this paper.

Another feature shown by Figure 5 is that in addition to the decline in output growth, there is also an increase in the volatility of growth. This is shown by the widening band between the 5th and 95th percentiles for the 1,000 repeated simulations.

4 Spill over of crises to emerging countries

The current account surplus experienced by emerging countries before the financial crisis, allowed these countries to accumulate financial assets issued

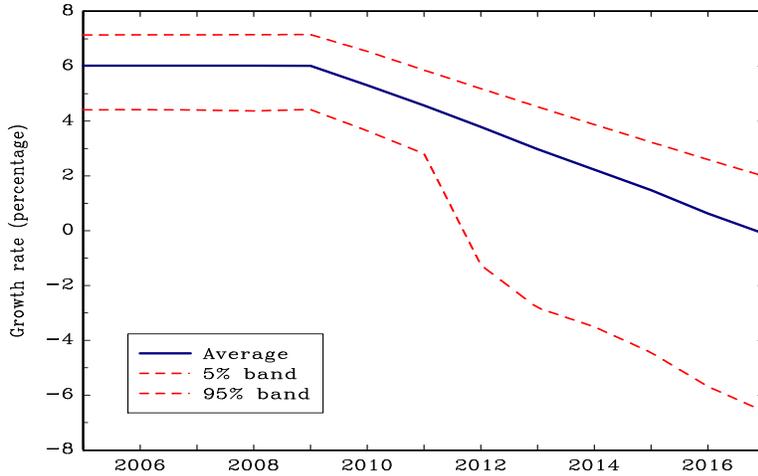


Figure 5: Response to low interest rates in Industrialized Countries, 2005-2017. Model with endogenous growth.

by industrialized countries. These assets, however, lost significant market value with the arrival of the financial crisis in industrialized countries, which translated in significant capital losses for emerging countries. An example is given by mortgage-based securities sold to investors in many countries including emerging economies. Using the model we can now explore how the capital losses experienced by emerging countries affected the macroeconomic performance of these economies.

Figure 6 shows what would happen to the growth rate of output if the value of foreign assets held by emerging countries were to drop by 50 percent in 2008 as a consequence of the financial crisis. The left panels assume that the foreign interest rate does not change (industrial countries do not react to the crisis by changing monetary policy). The right panels, instead, assume that there is also a change in monetary policy in industrialized countries.

The foreign crisis and the associated capital losses lead to a decline in the growth rate of output which is quite persistent. This is because it takes a long time for entrepreneurs to rebuilt their wealth through savings. However, without a reduction in the foreign interest rate, the growth rate recovers over time (although slowly). With loose monetary policy, instead, the growth rate of output continues to drop after the crisis.

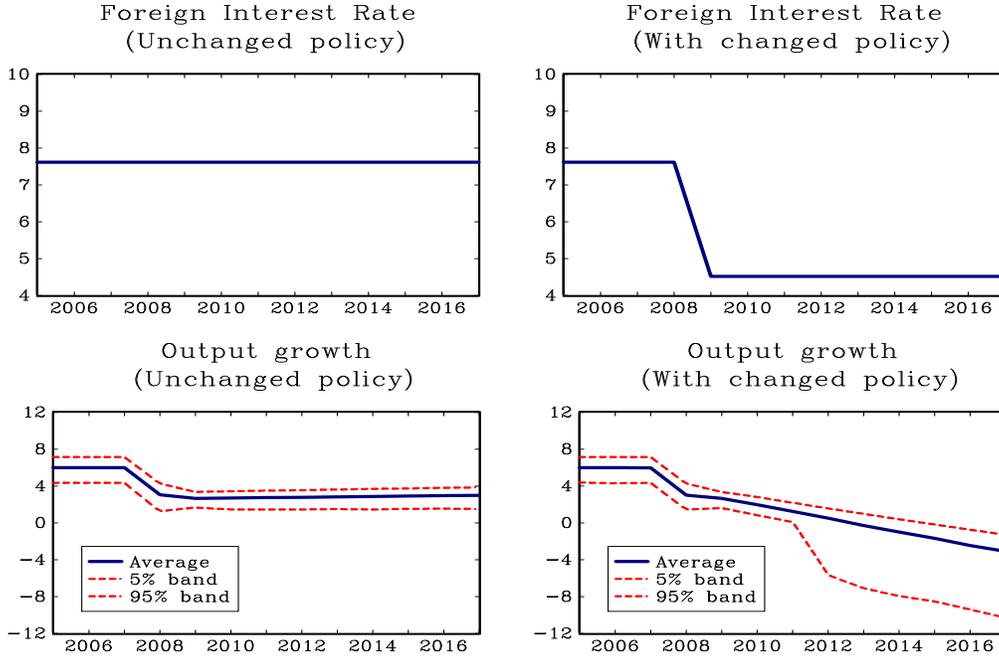


Figure 6: Response to foreign financial crisis leading to a loss in f_t in 2008.

5 Conclusion

In this paper I have shown that the low interest rate policies adopted by industrialized countries may have impacted negatively on the economic performance of emerging countries. Although lower interest rates in industrialized countries may have reduced the net outflow of capital away from emerging countries, lower interest rates also reduce savings, a channel ignored by the conventional view about the impact on monetary policy.

Although the theory proposed in this paper emphasizes the negative consequences of capital inflows, this should not be interpreted as suggesting that capital controls might be desirable. In this paper I only showed that capital inflows could have negative consequences for macroeconomic stability and growth if the inflows are caused by external factors. In particular, I focused on external monetary policy. However, if the inflows are driven by higher growth prospects for emerging countries, the inflows could be beneficial as they speed up the growth of these countries (by funding faster accumulation of capital). As far as the post-crisis period is concerned, however, it does

not appear that the inflows (or lower outflows) to emerging countries were caused by higher growth prospects, at least not ex-post.

Appendix

A Proof of Lemma 1.1

Ignoring the agent superscript i , the optimization problem of an entrepreneur can be written recursively as

$$\begin{aligned} \Omega_t(f_t, b_t) &= \max_{h_t} \mathbb{E}_t \tilde{\Omega}_t(a_t) & (12) \\ &\text{subject to} \\ &a_t = f_t + \tilde{b}_t + (z_t - w_t)h_t \\ &\tilde{b}_t = (1 - \delta_t)b_t \end{aligned}$$

$$\begin{aligned} \tilde{\Omega}_t(a_t) &= \max_{f_{t+1}, b_{t+1}} \left\{ \ln(c_t) + \beta \mathbb{E}_t \Omega_{t+1}(f_{t+1}, b_{t+1}) \right\} & (13) \\ &\text{subject to} \\ &c_t = a_t - q_t^f f_{t+1} - q_t^b b_{t+1} \end{aligned}$$

Since the information set changes from the beginning of the period to the end of the period, the optimization problem has been separated according to the available information. In sub-problem (12) the entrepreneur chooses the input of labor before knowing the productivity z_t . The variable δ_t is an aggregate stochastic variable that denotes the possible losses incurred by the entrepreneur at the beginning of the period. This is taken as given by an individual entrepreneur. In sub-problem (13) the entrepreneur allocates the end of period wealth in consumption and savings after observing z_t .

The first order condition for sub-problem (12) is

$$\mathbb{E}_t \frac{\partial \tilde{\Omega}_t}{\partial a_t} (z_t - w_t) = 0.$$

The envelope condition from sub-problem (13) gives

$$\frac{\partial \tilde{\Omega}_t}{\partial a_t} = \frac{1}{c_t}.$$

Substituting in the first order condition we obtain

$$\mathbb{E}_t \left(\frac{z_t - w_t}{c_t} \right) = 0. \quad (14)$$

At this point we proceed by guessing and verifying the optimal policies for employment and savings. The guessed policies take the form:

$$h_t = \phi_t(f_t + \tilde{b}_t) \quad (15)$$

$$c_t = (1 - \beta)a_t \quad (16)$$

$$q_t^b b_{t+1} = \theta_t \beta a_t \quad (17)$$

$$q_t^f f_{t+1} = (1 - \theta_t) \beta a_t \quad (18)$$

Since $a_t = f_t + \tilde{b}_t + (z_t - w_t)h_t$ and the employment policy is $h_t = \phi_t(f_t + \tilde{b}_t)$, the end of period wealth can be written as $a_t = [1 + (z_t - w_t)\phi_t](f_t + \tilde{b}_t)$. Substituting the guessed consumption policy we obtain

$$c_t = (1 - \beta) \left[1 + (z_t - w_t)\phi_t \right] (f_t + \tilde{b}_t). \quad (19)$$

This expression is used to replace c_t in the first order condition (14) to obtain

$$\mathbb{E}_t \left[\frac{z_t - w_t}{1 + (z_t - w_t)\phi_t} \right] = 0, \quad (20)$$

which is the condition stated in Lemma 1.1.

To complete the proof, we need to show that the guessed policies (15) and (16) satisfy the optimality condition for the choice of consumption and saving. This is characterized by the first order conditions of sub-problem (13), which is equal to

$$\begin{aligned} -\frac{q_t^f}{c_t} + \beta \mathbb{E}_t \frac{\partial \Omega_{t+1}}{\partial f_{t+1}} &= 0, \\ -\frac{q_t^b}{c_t} + \beta \mathbb{E}_t \frac{\partial \Omega_{t+1}}{\partial b_{t+1}} &= 0. \end{aligned}$$

From sub-problem (12) we derive the envelope conditions

$\partial \Omega_t / \partial f_t = 1/c_t$ and $\partial \Omega_t / \partial b_t = \mathbb{E}_t[(1 - \delta_t)/c_t]$ which can be used in the first order conditions to obtain

$$\begin{aligned} \frac{q_t^f}{c_t} &= \beta \mathbb{E}_t \frac{1}{c_{t+1}}, \\ \frac{q_t^b}{c_t} &= \beta \mathbb{E}_t \frac{1 - \delta_{t+1}}{c_{t+1}}. \end{aligned}$$

We have to verify that the guessed policies satisfy this condition. Using the guessed policy (16) and equation (19) updated one period, the first order conditions

can be rewritten as

$$\frac{q_t^f}{a_t} = \beta \mathbb{E}_t \frac{1}{[1 + (z_{t+1} - w_{t+1})\phi_{t+1}](f_{t+1} + \tilde{b}_{t+1})},$$

$$\frac{q_t^b}{a_t} = \beta \mathbb{E}_t \frac{1 - \delta_{t+1}}{[1 + (z_{t+1} - w_{t+1})\phi_{t+1}](f_{t+1} + \tilde{b}_{t+1})}.$$

Since z_{t+1} is independent of δ_{t+1} and \tilde{b}_{t+1} , the first order conditions can be rewritten as

$$\frac{q_t^f}{a_t} = \beta \mathbb{E}_t \left\{ \frac{1}{1 + (z_{t+1} - w_{t+1})\phi_{t+1}} \right\} \mathbb{E}_t \left\{ \frac{1}{f_{t+1} + \tilde{b}_{t+1}} \right\},$$

$$\frac{q_t^b}{a_t} = \beta \mathbb{E}_t \left\{ \frac{1}{1 + (z_{t+1} - w_{t+1})\phi_{t+1}} \right\} \mathbb{E}_t \left\{ \frac{1 - \delta_{t+1}}{f_{t+1} + \tilde{b}_{t+1}} \right\}.$$

Condition (20) implies that the first term on the right-hand-side is 1. Therefore, we can rewrite the first order conditions as

$$\frac{q_t^f}{\beta a_t} = \mathbb{E}_t \left\{ \frac{1}{f_{t+1} + \tilde{b}_{t+1}} \right\},$$

$$\frac{q_t^b}{\beta a_t} = \mathbb{E}_t \left\{ \frac{1 - \delta_{t+1}}{f_{t+1} + \tilde{b}_{t+1}} \right\}.$$

Now we can use $q_t^f f_{t+1} = (1 - \theta)\beta a_t$ and $q_t^b b_{t+1} = \theta\beta a_t$ in the two conditions to obtain

$$\mathbb{E}_t \left\{ \frac{q_t^b}{(1 - \delta_{t+1})q_t^f \theta + q_t^b(1 - \theta)} \right\} = 1,$$

$$\mathbb{E}_t \left\{ \frac{(1 - \delta_{t+1})q_t^f}{(1 - \delta_{t+1})q_t^f \theta + q_t^b(1 - \theta)} \right\} = 1,$$

where the first equation corresponds to the second condition reported in Lemma 1.1. *Q.E.D.*

B First order conditions for households

The optimization problem of a household is

$$W_t(l_t, k_t) = \max_{h_{t+1}, l_{t+1}, k_{t+1}} \left\{ c_t - A_t h_t + \beta \mathbb{E}_t W_{t+1}(l_{t+1}, k_{t+1}) \right\}$$

subject to

$$c_t = \frac{\mathbb{E}_t \tilde{l}(l_{t+1}, \tilde{p}_{t+1} k_{t+1})}{\bar{R}_t} + w_t h_t + (A_t + p_t) k_t - \tilde{l}(l_t, \tilde{p}_t k_t) - \varphi \left(\frac{l_t}{\tilde{p}_t k_t} \right) l_t - p_t k_{t+1}.$$

The first order conditions with respect to h_t , l_{t+1} , k_{t+1} are, respectively,

$$w_t = A_t,$$

$$\frac{1}{\bar{R}_t} \frac{\partial \mathbb{E} \tilde{l}(l_{t+1}, \tilde{p}_{t+1} k_{t+1})}{\partial l_{t+1}} + \beta \mathbb{E}_t \frac{\partial W_{t+1}(l_{t+1}, k_{t+1})}{\partial l_{t+1}} = 0,$$

$$\frac{1}{\bar{R}_t} \frac{\partial \mathbb{E} \tilde{l}(k_{t+1}, \tilde{p}_{t+1} k_{t+1})}{\partial k_{t+1}} - p_t + \beta \mathbb{E}_t \frac{\partial W_{t+1}(l_{t+1}, k_{t+1})}{\partial k_{t+1}} = 0.$$

The envelope conditions are

$$\frac{\partial W_t(l_t, k_t)}{\partial l_t} = - \frac{\partial \tilde{l}(l_t, \tilde{p}_t k_t)}{\partial l_t} - \frac{\partial \varphi \left(\frac{l_t}{\tilde{p}_t k_t} \right)}{\partial l_t} l_t - \varphi \left(\frac{l_t}{\tilde{p}_t k_t} \right),$$

$$\frac{\partial W_t(l_t, k_t)}{\partial k_t} = A_t + p_t - \frac{\partial \tilde{l}(l_t, \tilde{p}_t k_t)}{\partial k_t} - \frac{\partial \varphi \left(\frac{l_t}{\tilde{p}_t k_t} \right)}{\partial k_t} l_t.$$

Updating by one period and substituting in the first order conditions for l_{t+1} and k_{t+1} we obtain

$$\frac{1}{\bar{R}_t} = \beta \left[1 + \frac{\frac{\partial \mathbb{E} \varphi \left(\frac{l_{t+1}}{\tilde{p}_{t+1} k_{t+1}} \right) l_{t+1}}{\partial l_{t+1}}}{\frac{\partial \mathbb{E} \tilde{l}(l_{t+1}, \tilde{p}_{t+1} k_{t+1})}{\partial l_{t+1}}} + \frac{\mathbb{E} \varphi \left(\frac{l_{t+1}}{\tilde{p}_{t+1} k_{t+1}} \right)}{\frac{\partial \mathbb{E} \tilde{l}(l_{t+1}, \tilde{p}_{t+1} k_{t+1})}{\partial l_{t+1}}} \right] \quad (21)$$

$$p_t = \beta \mathbb{E} (A_{t+1} + p_{t+1}) + \beta \left[\left(\frac{1}{\beta \bar{R}_t} - 1 \right) \frac{\partial \mathbb{E} \tilde{l}(l_{t+1}, \tilde{p}_{t+1} k_{t+1})}{\partial k_{t+1}} - \frac{\mathbb{E} \varphi \left(\frac{l_{t+1}}{\tilde{p}_{t+1} k_{t+1}} \right)}{\partial k_{t+1}} l_{t+1} \right] \quad (22)$$

Let's focus on $\tilde{l}(l_{t+1}, \tilde{p}_{t+1}k_{t+1})$ and $\varphi(l_{t+1}/\tilde{p}_{t+1}k_{t+1})$ defined in (3) and (4). Assuming that the optimal choice of l_{t+1} and k_{t+1} satisfy $l_{t+1} < p_{t+1}k_{t+1}$ and $l_{t+1} > \xi A_t k_{t+1}$, we have

$$\begin{aligned}\frac{\partial \mathbb{E}\tilde{l}(l_{t+1}, \tilde{p}_{t+1}k_{t+1})}{\partial l_{t+1}} &= 1 - \lambda \\ \frac{\partial \mathbb{E}\tilde{l}(l_{t+1}, \tilde{p}_{t+1}k_{t+1})}{\partial k_{t+1}} &= \lambda \xi A_t \\ \mathbb{E}\varphi(l_{t+1}/\tilde{p}_{t+1}k_{t+1}) &= \lambda \chi \left(1 - \frac{\xi A_t k_{t+1}}{l_{t+1}}\right)^2 \\ \frac{\partial \mathbb{E}\varphi(l_{t+1}/\tilde{p}_{t+1}k_{t+1})}{\partial l_{t+1}} l_{t+1} &= 2\lambda \chi \left(1 - \frac{\xi A_t k_{t+1}}{l_{t+1}}\right) \frac{\xi A_t k_{t+1}}{l_{t+1}} \\ \frac{\partial \mathbb{E}\varphi(l_{t+1}/\tilde{p}_{t+1}k_{t+1})}{\partial k_{t+1}} l_{t+1} &= -2\lambda \chi \left(1 - \frac{\xi A_t k_{t+1}}{l_{t+1}}\right) \xi A_t\end{aligned}$$

We can see that the first two terms do not depend on l_{t+1} and k_{t+1} , while the last three terms are functions of the ratio l_{t+1}/k_{t+1} . Therefore, we can express the first order conditions (21) and (22) as,

$$\frac{1}{\bar{R}_t} = \beta + \Phi_t \left(\frac{l_{t+1}}{k_{t+1}} \right) \quad (23)$$

$$p_t = \beta \mathbb{E} \left(A_{t+1} + p_{t+1} \right) + \Psi_t \left(\frac{l_{t+1}}{k_{t+1}} \right). \quad (24)$$

It can be verified that the functions $\Phi_t(\cdot)$ and $\Psi_t(\cdot)$ are both increasing for $l_{t+1}/k_{t+1} > \xi A_t$. The time subscript takes into account the dependence on the aggregate state A_t . Conditions (23) and (24) are the equivalent of (7) and (8). *Q.E.D.*

C Market for liquidated houses

The functioning of the market for liquidated houses is characterized by two assumptions.

Assumption 1 *Houses can be sold either to other domestic households or domestic entrepreneurs. If sold to entrepreneurs, houses lose their functionality and must be converted to consumption goods at rate ξA_t .*

This assumption formalizes the idea that houses may lose value when reallocated to owners that do not use them directly. In the model this is proxied by

assuming that entrepreneurs convert houses in consumption goods at rate ξA_t , which is typically lower than the price of houses in normal times.⁵

The parameter ξ determines the liquidation price of houses when the housing market freezes. It is important to point out that, in order for houses not to lose their functionality, they need to be purchased by domestic households. The question is then whether households have the capability of purchasing liquidated houses. This is established in the next assumption.

Assumption 2 *Households can purchase liquidated houses only if $l_t < \tilde{p}_t k_t$.*

If a household starts with liabilities that are bigger than the liquidation value of its own house, that is, $l_t > \tilde{p}_t k_t$, the household will be unable to raise additional funds to purchase the liquidated houses of other households. Potential lenders know that the new loan (as well as the outstanding liabilities) is not collateralized and the household will renegotiate immediately after taking the new loan. I refer to a household for which $l_t > \tilde{p}_t k_t$ as ‘illiquid’ since it cannot raise any funds.

To better understand Assumptions 1 and 2, consider the condition for not renegotiating, $l_t \leq p_t k_t$. Furthermore, assume that $p_t > \xi A_t$, that is the price of houses in normal time, p_t is bigger than the value of houses for entrepreneurs. If this condition is satisfied, households have the ability to raise funds to purchase the house of a defaulting household. This insures that the market price for the liquidated house is p_t . However, if $l_t > \xi A_t k_t$ for all households, there will be no household capable of participating in the market. As a result, the liquidated house can only be sold to entrepreneurs at price $\tilde{p}_t = \xi A_t$.

This shows that the value of liquidated houses depends on the financial decision of households, which in turn depends on the price. This interdependence creates the conditions for multiple self-fulfilling equilibria.

Proposition C.1 *There exists multiple equilibria only if $l_t > \xi A_t$.*

When multiple equilibria are possible, the equilibrium is selected through the random draw of sunspot shocks.

Let ε_t be a variable that takes the value of 0 with probability λ and 1 with probability $1 - \lambda$. If the condition for multiplicity is satisfied, agents coordinate their expectations on the low liquidation price $\tilde{p}_t = \xi A_t$ when $\varepsilon_t = 0$. Thus, the probability distribution of the low liquidation price is

$$\Upsilon_{t-1}(\tilde{p}_t = \xi A_t) = \begin{cases} 0, & \text{if } l_t \leq \xi A_t k_t \\ \lambda, & \text{if } \xi A_t k_t < l_t \end{cases}$$

⁵Since the supply of houses \bar{K} is fixed, while the services from houses depend on productivity, the price of houses grows with productivity.

If leverage is sufficiently small ($l_t/\xi A_t k_t < 1$), households remain liquid even if the (expected) liquidation price is ξA_t . But then the liquidation price cannot be low and the realization of the sunspot shock is irrelevant for the equilibrium. Instead, when the leverage is high, the liquidity of households depends on the price. In this case the realization of ε_t becomes important for selecting one of the two equilibria. When $\varepsilon_t = 0$ —which happens with probability λ —the market expects the liquidation price to be ξA_t , making the household's sector illiquid. On the other hand, when $\varepsilon_t = 1$ —which happens with probability $1 - \lambda$ —the market expects the liquidation price to be the one that prevails in the market with the participation of households, validating the expectation of the high liquidation price.⁶ If the leverage is very large, however, households are always illiquid and the equilibrium price is ξA_t .

Notice that the argument is based on the assumption that ξ is sufficiently low (implying that $\xi A_t < p_t$). Also, the equilibrium value of houses at the end of the period, $p_t k_t$, is always bigger than the debt, l_t . Condition (5) determining the interest rate guarantees that this will always be satisfied in equilibrium. Furthermore, assuming that l_t is always bigger than ξA_t , the liquidation price \tilde{p}_t fluctuates between p_t and ξA_t as assumed in (2).

⁶The assumption that houses lose their functionality if sold to foreign households, in addition to entrepreneurs, allows me to have equilibrium in which the default happens only in one country. If houses could maintain their functionality when sold to foreign entrepreneurs, implies that default in one country could arise only if the other country also defaults. Nevertheless, even if default takes place only in one country, we will see that it impacts the macro-economy of the other country because of the portfolio diversification of entrepreneurs.

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