Large Shareholders and Financial Distress

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Abstract

I examine large shareholders’ externalities on other claimholders when firms are in financial distress. To this end, I develop a tractable dynamic credit risk model featuring the interaction between blockholders and other investors. Blockholders’ information acquisition and funding decisions play a pivotal role in distressed firms’ access to finance, affecting both total firm value and its distribution across claims. The impact on distress costs is generically non-monotone — whereas blockholders exacerbate inefficiencies for intermediate levels of distress, they alleviate costs in deep distress. The results reveal that frictions that delay block acquisitions to “last minute” rescue interventions can in fact be efficiency-enhancing.

Keywords: Blockholders, Financial Distress Costs, Debt Overhang, Credit Risk, Private Investments in Public Equity

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1. Introduction

Large shareholdings by wealthy individuals and financial institutions are a pervasive empirical phenomenon. According to Holderness (2009), 96% of US firms contain at least one blockholder defined as a shareholder who holds more than 5% of outstanding shares. A key channel differentiating blockholders from small shareholders is that large exposures alleviate free-rider problems among investors (see, e.g., Grossman and Hart, 1980, Shleifer and Vishny, 1986). In financial distress, these free-rider problems can be particularly consequential, as they can culminate in default — lacking sufficient incentives to acquire information about the firm’s underlying solvency, each individual investor may refuse to provide funds when doing so is crucial for the firm’s survival. In contrast, blockholders have substantial incentives to stay informed, implying that they are better equipped to distinguish solvent from insolvent firms and are willing to provide support when fundamentals are in fact solid. Mirroring these advantages, firms in practice indeed largely rely on new or old blockholders when raising external finance in financial distress (Park, 2011). At the same time, conflicts of interest between debt and equity holders are usually the most severe for highly indebted firms,

In this paper, I examine blockholders’ externalities in financial distress to shed light on these advantages and potential sources of inefficiencies. I develop a tractable dynamic credit risk model featuring blockholders whose funding decisions can play a pivotal role in distressed firms’ access to finance. Regular investors optimally follow a blockholder’s lead in that they infer information from her willingness to provide continued support to the firm. The large shareholder’s information acquisition and funding decisions thus create an externality by affecting the information available to other investors. This externality not only influences other equity holders’ claim values, but also implies non-monotone effects for default risk, debt values, and

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1See Jensen and Meckling (1976) and Myers (1977).
overall firm value.

Compared to risk shifting — which *increases* the volatility of asset cash flows through asset substitution (as in Jensen and Meckling, 1976) — blockholders’ informational externality *reduces* investors’ conditional uncertainty about asset cash flows. Yet, it also raises the volatility of posterior beliefs, and thus, the volatility of valuations of those cash flows. This effect has first-order implications for equity holders’ strategic default decisions, which govern the risk of bankruptcy. As in standard debt overhang problems, equity holders’ decisions in the model are generally socially inefficient, because shareholders do not internalize the benefits of continuation investments that accrue to existing debt holders.

The model reveals a non-monotone impact of blockholders on these inefficiencies as a function of the severity of distress — whereas blockholders amplify inefficiencies for low to medium levels of distress, they generically alleviate them in deep distress, in case of “last minute” rescue interventions. The latter effect is well-illustrated by considering a firm that is destined to default at the next debt payment unless investors significantly revise their beliefs upwards. Here, a blockholder can only do good by providing a chance of resurrection; if the blockholder uncovers negative information, default is merely confirmed. Yet, when she uncovers positive information and chooses to inject more funds, the firm survives at the next payment date. In contrast, when a firm is in a stronger position, blockholders’ investigation efforts increase default risk over horizons such as a year. Firms that would likely survive absent blockholders’ informational externality now face the hazard that a blockholder will pull her support, signaling the firm’s underlying weakness and, in turn, causing default. Since default tends to lead to greater inefficiencies in recessions (e.g., due to greater frictions in reallocating capital; see Caballero and Hammour, 2005, Eisfeldt and Rampini, 2006), any changes in default rates have a more material effect on overall firm value in these times. As a result, blockholders’ positive impact on deeply distressed corporations’ overall firm value can be quite substantial in recessions. In contrast, in booms, the effects are dampened by smaller inefficiencies associated with
default.

Large shareholders’ impact on debt values is a priori theoretically ambiguous. On the one hand, large shareholders’ information facilitates the optimal exercising of equity holders’ put option, in which debt investors hold a short position. That is, large shareholders’ increased attention to firm conditions generates a redistribution of value from debt to equity holders. On the other hand, the total value to be distributed between debt and equity holders — that is, firm value — may increase, since default risk may decline. Incorporating these competing effects, the calibrated model predicts that in booms, debt value changes exhibit an inverse hump-shaped pattern as a function of the interest coverage ratio. In these states, debt values are consistently negatively affected by the actions of the large shareholder. In contrast, in recessions, debt values are positively affected for deeply distressed firms with uncertain growth prospects, as large shareholders tend to reduce default risk for these firms. Moreover, as default leads to greater deadweight losses in recessions, these reductions in default risk have a larger positive effect on overall firm value. In net, a large shareholder can therefore positively affect debt values in highly distressed states, in particular when the deadweight losses from bankruptcy are material.

In practice, large investors often acquire substantial stakes in distressed firms’ private placements of public equity (PIPE transactions). In the model, existing shareholders are willing to issue new equity to large investors at a discounted price when the firm requires new funds and existing shareholders are uncertain about firm solvency. Despite the discounted offer, involving a large investor can be in the interest of existing shareholders, since atomistic investors anticipate the ability to free-ride on the large shareholder’s information acquisition efforts going forward. The model endogenizes the incidence of PIPE transactions and reveals that these transactions are more likely to occur for low interest coverage ratios and in states with substantial uncertainty about the firm’s future growth prospects, mirroring the findings of the empirical lit-
erature. Interestingly, matching and negotiation frictions causing delays in large shareholders’ involvement can have positive implications for overall firm value. The efficiency-enhancing effects of large shareholders’ actions dominate the value-destroying rent-seeking effects for “last minute” interventions, that is, interventions in states in which the firm is about to file for bankruptcy unless a large investor rescues the firm. These results suggest that frictions causing these transactions to predominantly occur in the form of such last minute interventions can have positive implications for overall efficiency.

On the modeling side, the proposed environment nests canonical quantitative credit risk models but generically features lumpiness in both debt payments and earnings adjustments. This modeling approach provides increased tractability, which is particularly useful when introducing large investors and endogenous information acquisition. The model yields distributions and valuations in closed-form for any given policy function, and allows obtaining global solutions merely by inverting sparse matrices. Global solutions, in turn, are essential to accurately examining large shareholders’ highly non-linear effects close to the default boundaries. Moreover, by nesting standard quantitative credit risk models and yielding solutions for conditional distributions, the model lends itself to both qualitative and quantitative analyses. Counterfactual analyses based on the calibrated model help overcome some of the identification challenges of typical reduced-form approaches examining blockholders’ externalities, such as cross-sectional regressions or event studies based on announcement returns.

Following Grossman and Hart (1980) and Shleifer and Vishny (1986), an influential theoretical literature has analyzed channels through which large shareholders affect corporate governance. In these studies, blockholders overcome free-rider problems and can alleviate conflicts of interest between shareholders and management (see, e.g., Maug, 1998, Edmans and Manso, 2011) but may also seek private benefits and harm small shareholders (see, e.g., Burkart, Gromb,

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2Brophy, Ouimet, and Sialm (2009) document that hedge funds tend to finance companies that have poor fundamentals and pronounced informational frictions, and require substantial discounts.
and Panunzi, 1997, Pagano and Rell, 1998). ³ Contrary to this literature, I study large shareholders’ externalities in financial distress, when conflicts of interest between debt and equity holders are the most severe. My analysis focuses on these conflicts of interest rather than conflicts of interest between equity holders and management, which are not specific to financial distress. In fact, by having a disciplining effect on managers, high levels of leverage are typically seen as a channel mitigating or eliminating the agency costs of free cash flow (Jensen, 1986). In my analysis, blockholders affect other investors’ willingness to support the firm, which is needed to avert bankruptcy and associated inefficiencies. Through this channel, blockholders effectively shape the severity of debt overhang problems distorting efficient continuation investments. Moreover, my analysis is based on a structural credit risk model that lends itself to both qualitative and quantitative analyses.

In using a structural approach and considering conflicts of interest between debt and equity holders my paper is related to analyses of debt overhang problems (see, e.g., Moyen, 2007, Hackbarth, 2009) and of credit risk in dynamic environments of the Leland (1994) tradition. I contribute to this literature by analyzing the effects of large shareholders and their endogenous information acquisition. These elements also differentiate my paper from the existing literature on the relationship between capital structure and asset pricing (e.g., Hackbarth, Miao, and Morellec, 2006, Livdan, Sapirza, and Zhang, 2009, Gomes and Schmid, 2010). Following the structural credit risk models of Chen (2010) and Bhamra, Kuehn, and Strebulaev (2010), my model features business cycle fluctuations that play a quantitatively important role for valuations and default costs.

In addition, several papers in the credit risk literature feature learning, but do not consider blockholders. Duffie and Lando (2001) study the implications of imperfect information for

³See Edmans (2014) for a comprehensive review of theoretical and empirical papers in this literature. See also Brav, Jiang, Partnoy, and Thomas (2008) for evidence on channels through which large investors may affect conflicts of interest between shareholders and management, including proxy fights and shareholder proposals to replace management.
credit spreads, assuming that bond investors cannot observe information available to equity holders directly, and receive instead only periodic and imperfect accounting reports. David (2008) features learning about aggregate dynamics and assumes the same default rules as in Merton (1974) in order to solve for prices. This assumption implies that a key channel of my paper — that optimal default decisions (and debt overhang) depend on the speed with which information is expected to arrive in the future (e.g., from a blockholder) — is absent. Similarly, this channel is absent in Johnson’s (2004) analysis of a Merton (1974) type setting with noise.

2. The Model

The economy is in continuous time. A levered firm with perpetual debt obligations either has an existing legacy blockholder, or may engage in a transaction with a new large investor to establish a block position. Below, I first describe the processes governing aggregate dynamics. Afterwards, I detail the market structure, technology, and the objectives of firms and large investors.

2.1. Aggregate Processes and Stochastic Discount Factor

Macroeconomic conditions. The existing structural credit risk literature has highlighted that the pricing of persistent macroeconomic fluctuations significantly improves models’ ability to capture central empirical facts about the pricing of debt claims (see, e.g., Bhamra, Kuehn, and Strebulav, 2010, Chen, 2010). As my paper aims to provide both qualitative and quantitative predictions, I allow for the presence of such persistent fluctuations. Let $Z$ denote an aggregate state that governs persistent, mean-reverting variation in the macroeconomic environment (e.g., booms vs. recessions). $Z$ follows a continuous-time Markov chain that takes values in the discrete set $\Omega_Z$ and has the generator matrix $\Lambda_Z$, which collects transition rates between states.
\[ Z \in \Omega_Z. \] Let \( N_{Z,t} \) denote a matrix that collects counting processes that keep track of all jumps between the states \( Z \in \Omega_Z \). To balance increases in the off-diagonal elements, the diagonal elements of this matrix count down by one each time a given state \( Z \) is left for another state \( Z' \).

I also define the matrix \( M_{Z,t} \) collecting the compensated processes:

\[
dM_{Z,t} = dN_{Z,t} - \Lambda_Z dt. \tag{1}
\]

Throughout the paper, for any given matrix \( M \), I will use the notation \( M(x) \) to indicate the \( x \)-th row of that matrix, and \( M(x, x') \) to represent the \( (x, x') \)-th element.

**Aggregate trend.** The state \( Y \) captures an aggregate trend in the economy that follows a geometric Brownian motion:

\[
\frac{dY_t}{Y_t} = \mu_Y(Z_t)dt + \sigma_Y(Z_t)dB_t. \tag{2}
\]

Agents can observe the aggregate states \( Z \) and \( Y \).

**SDF dynamics.** Let \( m \) denote the stochastic discount factor (SDF) reflecting agents’ marginal utility. I specify a flexible process for \( m \) that can capture the pricing properties of a variety of benchmark asset pricing models. The SDF follows a Markov-modulated jump diffusion process:

\[
\frac{dm(S_t)}{m(S_t)} = -r_f(Z_t) dt - \nu(Z_t) dB_t + \sum_{Z' \neq Z_t} (e^{\phi(Z_t, Z')} - 1) dM_{Z,t} (Z_t, Z'), \tag{3}
\]

where \( r_f \) denotes the risk free rate, \( \nu \) is the price of risk for aggregate Brownian shocks, and \( \phi(Z, Z') \) determines the jumps in \( m \) conditional on a change in the state \( Z \). Let \( \bar{\Lambda}_Z \) denote the generator matrix under the risk neutral measure, which collects the risk neutral transition rates \( \bar{\Lambda}_Z(Z, Z') = e^{\phi(Z, Z')} \Lambda_Z(Z, Z') \).
2.2. The Firm

As commonly assumed in the structural credit risk literature, the earnings of a firm are split between contractual debt payments promised to debt holders and a dividend paid to equity holders, that is, the firm does not hold excess cash (see, e.g., Goldstein, Ju, and Leland, 2001, Hackbarth, Hennessy, and Leland, 2007, Strebulaev, 2007, Bhamra, Kuehn, and Strebulaev, 2010, Chen, 2010).\(^4\) Throughout, management acts in the interest of existing shareholders. The corporate tax rate is denoted by \(\tau\).

**Earnings dynamics.** Let \(X_t \geq 0\) denote the firm’s before-interest earnings rate. I assume that \(X_t = Y_t \cdot e^{x_t}\), that is, earnings are affected linearly by the aggregate trend factor \(Y_t\) and by a firm specific state \(x_t\). Detrended log-earnings \(x_t\) take values in a discrete set \(\Omega_x\), the elements of which constitute an equidistant grid with increments of size \(\Delta_x > 0\).\(^5\) By choosing \(\Delta_x\) small enough, this specification can approximate a continuous support arbitrarily well.\(^6\) Moreover, the discrete state space structure allows capturing lumpy earnings adjustments (e.g., as caused by new sales transactions in practice).

Let \(N_{x,t}^+\) and \(N_{x,t}^-\) denote Poisson processes keeping track of the number of upward and downward innovations to detrended earnings. The corresponding evolution equation for \(x\) is given by:

\[
dx_t = \Delta_x \cdot (dN_{x,t}^+ - dN_{x,t}^-). \tag{4}
\]

\(^4\)Apart from tax implications, cash holdings would not affect the analysis if the firm could pay out cash at any time before default is triggered.

\(^5\)The setup can also easily accommodate grids of detrended log-earnings that are non-equidistant.

\(^6\)Models with a continuous support are in any case approximated by a discrete support when solved numerically.
The earnings rate $X_t$ then follows the jump-diffusion process:

$$
\frac{dX_t}{X_t} = \mu_Y(Z_t)dt + \sigma_Y(Z_t)dB_t + (e^{\Delta x} - 1)dN_{x,t}^+ + (e^{-\Delta x} - 1)dN_{x,t}^-.
$$

As further discussed below, this specification nests as a limiting case the standard specification in the literature that earnings follow a geometric Brownian motion.

At any point in time $t \geq 0$, the arrival intensities of innovations to detrended log-earnings $x$ are affected by the firm-specific state $\theta_t \in \Omega \theta = \{g, b\}$. The possible $\theta$-values $g$ and $b$ refer to a good state and a bad state, respectively. The process for $\theta_t$ is specified as follows. Over time, the firm obtains observable shocks to the state $\theta_t$ with Poisson arrival rate $\lambda_\theta$. Upon these shocks, a new value for $\theta$ is drawn, with $g$ being drawn with probability $\hat{\pi}(Z)$, and $b$ being drawn with complimentary probability $(1 - \hat{\pi}(Z))$. Between these shocks, $\theta_t$ stays constant.

Let $N_{\theta,t}$ denote the Poisson process that keeps track of the number of observable shocks to the state $\theta_t$ since date 0. Even though agents can observe innovations to $N_{\theta,t}$ (that is, agents know when a shock occurred), they cannot directly observe the state $\theta_t$ itself, that is, $\theta_t$ is a hidden state. Agents are Bayesian learners and form rational beliefs about $\theta_t$ based on the history of all variables they can observe.

Let $\lambda^+_x(\theta, Z)$ and $\lambda^-_x(\theta, Z)$ denote the Poisson arrival rates of upward and downward innovations to detrended log-earnings $x$, respectively. It is convenient to impose the parameter restriction that the total Poisson arrival rate with which any adjustment to $x$ occurs (either an upward or a downward adjustment) does not depend on the hidden state $\theta$, although it may vary with the aggregate state $Z$. Specifically, the total jump intensity is defined as:

$$
\lambda_x(Z) \equiv \lambda^+_x(\theta, Z) + \lambda^-_x(\theta, Z) \quad \text{for all } \theta, Z.
$$

(6)
This specification implies that the mean and the volatility of $x$ are given by:

$$
\mu_x(\theta, Z) = \Delta_x(\lambda^+_x(\theta, Z) - \lambda^-_x(\theta, Z)),
$$

$$
\sigma_x(Z) = \Delta_x \sqrt{\lambda_x(Z)},
$$

that is, whereas the drift $\mu_x$ depends on the state $\theta_t$, the volatility $\sigma_x$ is independent of the hidden state. Imposing that the total arrival intensity $\lambda_x$ does not vary with the hidden state $\theta_t$ ensures that volatility does not immediately reveal the hidden state $\theta_t$ even in the limiting case where the $x$-process approaches a Brownian motion.\(^7\) Moreover, conditional on any choice for $\Delta_x$, all Poisson intensities governing the dynamics for $x$ are then uniquely pinned down by choosing the drifts $\mu_x(\theta, Z)$ and the volatilities $\sigma_x(Z)$. Correspondingly, I will calibrate the model by directly choosing values for $\mu_x(\theta, Z)$ and $\sigma_x(Z)$, and backing out the associated Poisson intensities.

**Lumpy debt obligations.** At time $t = 0$, the firm has existing debt obligations that involve lumpy payments to debt holders. Although it is straightforward to determine firms’ optimal ex ante leverage choices,\(^8\) my analysis will focus on blockholders’ impact in financial distress, that is, in states where firms already have excessive existing levels of leverage. For this reason, I will also abstract from additional debt issuances, which are suboptimal in these states, as the benefits of additional tax shields are outweighed by distress costs.

Payment dates associated with the existing debt obligations arrive with Poisson intensity $\lambda_C$ and require the firm to make lumpy payments of size $C_t = Y_t e^c$, where $c$ is a constant. That is, detrended debt payments are constant and equal to $e^c$. The expected rate of debt payments is

\(^7\)Suppose we specify, $\lambda_x(Z) = \frac{\sigma_x(Z)^2}{\Delta_x^2}$ for some fixed value $\sigma_x(Z)$. Then, for $\Delta_x \rightarrow 0$, the $x$-process approaches a Brownian motion with volatility $\sigma_x(Z)$.

\(^8\)See Appendix B.
then equal to $\lambda_C C_t$. It is convenient to introduce the state variable

$$\rho_t \equiv \log \left[ \frac{X_t}{\lambda_C C_t} \right],$$

(9)

which can be interpreted as the log interest coverage ratio. Let $N_{C,t}$ denote the Poisson process that keeps track of the number of debt payments since date 0, and let $\delta_t \in \{0, 1\}$ denote equity holders’ decision whether to make a debt payment at time $t$ if one comes due at that time. Upon default, debt holders recover a fraction $\alpha(Z)$ of the value of the firm’s unlevered assets.

It is worth noting that the tractability of the setup does not per se require that debt payments are lumpy. In fact, in the limiting case $\lambda_C \to \infty$, payments are made continuously. Lumpy debt payments are, however, a plausible feature of the data.

2.3. Blockholders

The modeling of large shareholders aims to capture the effects of two types of empirically relevant scenarios in financial distress. In the first scenario, the firm has a legacy blockholder that established its position a long time before the firm entered financial distress. For example, founder family members may be such large blockholders. In the second type of scenario, a new investor establishes a block exactly when the firm enters financial distress. These latter transactions typically involve specialized hedge funds or wealthy individuals that privately negotiate terms with management in “private placements of public equity.” Corresponding to these two types of scenarios, I will first lay out the setup with a legacy blockholder, and then describe how a new block may be established via a private placement of public equity.
2.3.1. Existing Blockholder

In the first scenario, the firm already has a legacy blockholder holding a fraction $\omega > 0$ of the firm’s equity. The blockholder maximizes the market value of her position accounting for the costs associated with information acquisition. When receiving assistance from the firm, the blockholder can produce incremental information on the firm’s hidden state $\theta$. Assistance in this context refers for example to the blockholder’s ability to discuss public information with the firm’s CFO (e.g., at a board meeting or at an earnings call) that helps the blockholder produce better estimates of the firm’s future earnings growth prospects. By incurring costs at a rate $I_t \geq 0$ the large shareholder’s investigation yields her a privately signal with a Poisson arrival rate $\lambda_A = \psi \bar{I}$, where $\psi > 0$, $\eta \in (0, 1)$, and $\bar{I} \equiv I/X$. To keep the analysis parsimonious, the signal is assumed to reveal the firm’s state $\theta_t$ without noise, although it is straightforward to relax this assumption. The specification for information acquisition costs implies cointegration with total firm size as measured by the variable $X$. I define $\{N_{A,t}\}_{t=0}^\infty$ as a counting process that keeps track of the number of signals generated since date 0.

Arrangement between the blockholder and the firm. The following arrangement determines whether a firm assists an existing blockholder’s information acquisition efforts. In the time period after the block was established, the firm asks the large shareholder to co-finance a fraction $\omega$ of any external funds needed to make debt payments (that is, whenever $dN_{C,t} = 1$). In return, the blockholder obtains a fraction $\omega$ of the newly issued shares. The blockholder has the option to reject these financing requests, but management is committed to refuse providing assistance to the large shareholder going forward after receiving such a rejection. Lacking the firm’s assistance, the blockholder then loses her ability to generate incremental information. As shown below, this arrangement will ensure truth-telling about private signals by the blockholder.

Let $\delta_{A,t} \in \{0, 1\}$ denote the large shareholder’s decision when asked to provide co-financing at time $t$. I introduce the firm-specific state variable $a_t \in \{0, 1\}$ that indicates whether a firm is
currently "matched" with a blockholder who receives assistance with information acquisition \((a = 1)\) or not \((a = 0)\).

**Trading environment in secondary markets.** The trading environment in secondary markets abstracts from noise traders (as, e.g., in Grossman and Stiglitz, 1980). As a result, all market participants understand that the blockholder trades only for informational reasons, she cannot extract additional information rents by trading against less informed investors (as in Milgrom and Stokey, 1982). Yet the blockholder will still have incentives to acquire information, as doing so improves her funding decisions and overall value.

Apart from maintaining focus and tractability, abstracting from noise is a plausible modeling choice in light of regulations in the United States that require investors acquiring more than 5% of a firm’s equity (with the intent to exert control) to file a schedule 13d with the SEC within 10 days. These investors have to re-file these forms in case of material changes to their positions (1%). These required public regulatory filings ensure that other investors learn about a blockholder’s position adjustments at a high enough frequency to inform their own decisions, which is the main channel the model aims to capture. Moreover, a blockholder unwinding her position of at least 5% would face price impact (see Kyle, 1989), resulting in information leakage even before the official regulatory filing takes place after 10 days. The resulting lack of incentives to trade in secondary markets implies that unless the blockholder rejects a funding request \((\delta_{A,t} = 0)\), she maintains an \(\omega\)-stake in the firm’s equity (until default occurs).

### 2.3.2. Establishing a New Block

The analysis also aims to shed light on the impact of private investments in public equity that take place when firms are in financial distress and choose to involve a new blockholder. A continuum of *large investors* may potentially assume this position. When a firm that currently is
not matched with a large investor has a payment date (when \( dN_{C,t} = 1 \)), one investor from this set is randomly chosen to obtain the opportunity to negotiate a transaction with the firm with probability \( \kappa \in (0, 1) \). Here, the parameter \( \kappa \) governs frictions in finding and matching with a firm. The larger these search frictions, the more does the market structure deviate from one with competitive free entry, which affects the magnitude of rents obtained by a large investor. If a negotiation opportunity is obtained, matching requires an investor to incur a fixed negotiation cost \( \chi_t = X_t \hat{\chi} \), capturing for example payments to lawyers that have to draft contracts. The presence of these fixed costs implies that transactions don’t just have upside for the parties involved — in equilibrium, a transaction will occur only if a large investor obtains a surplus greater than the fixed cost \( \chi_t \).

**Features of PIPE transactions.** The transactions negotiated between a large investor and a firm mirror standard private investments in public equity, which involve an investor purchasing a block of newly issued equity shares from a firm. In particular, the investor acquires an ownership share \( \omega \in (0, 1] \) at an endogenously determined purchase price \( P_t^A \).9 When negotiating the purchase price \( P_t^A \), the large investor is assumed to make a take-it-or-leave-it offer to management. When choosing whether to accept the offered price, management takes into account its outside option to remain a firm without a large shareholder and the possibility of matching with a different investor in the future. In these transactions negotiated with management, investors typically obtain substantial discounts relative to prevailing market prices (see, e.g., Brophy, Ouimet, and Sialm, 2009), and therefore, profits.

I specify the large investor’s ownership share \( \omega \) as an exogenous parameter. In practice, this ownership share is affected by regulations, a financial institution’s assets under management

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9As detailed above, I follow the common assumption in the literature that the firm does not hold excess cash (see, e.g., Goldstein, Ju, and Leland, 2001, Hackbarth, Hennessy, and Leland, 2007, Strebulaev, 2007, Bhamra, Kuehn, and Strebulaev, 2010, Chen, 2010). Thus, if the purchase price \( P_t^A \) is insufficient to cover the firm’s external cash needs when the transaction with the large investor occurs, other shareholders inject the remaining funds if doing so is optimal for them. Conversely, if the purchase price exceeds the cash needed (if \( \max(P_t^a - C_t, 0) > 0 \), the extra funds are settled with existing equity holders.
and investment mandate, and capital constraints, forces that are not in the focus of this paper. For example, NASDAQ Rule 5635(d) requires issuers to obtain prior approval of shareholders when an issuance below market value represents 20% or more of common shares.\textsuperscript{10} As a result of increased transaction costs and regulatory scrutiny above 20%, the empirical distribution of PIPE transactions involving financially distressed firms features significant bunching just below the regulatory 20% cutoff (see Park, 2011).

To summarize the information sets and the messages sent under this arrangement between management and the large investor: at the time of the initial transaction (when a match occurs), management and the investor have symmetric information, as the investor can obtain additional information only after establishing a match. As a result, bargaining under symmetric information obtains at the time of the initial transaction. On payment dates after the initial transaction, the less informed party (management) sends requests to the large shareholder to co-finance a fraction $\omega$ of external funds needed to make the debt payment. The large shareholder can respond to each request by either accepting or rejecting it.

### 3. Analysis

In this section, I analyze the model. I start by establishing how management and other investors optimally use the signals conveyed by the large shareholder’s co-financing decisions. Afterwards, I characterize the evolution of agents’ beliefs on the equilibrium path. Given this belief evolution, I solve for large investors’ optimal information investment and co-financing decisions, as well as for large shareholders’ value and the value of claims to the firm.

**Lemma 1** (Optimal default rule with a large shareholder). *In equilibrium, if a firm is matched with a large shareholder ($a_t = 1$), management chooses to trigger default if and only if the large

\textsuperscript{10}NYSE rule 312.03 and NYSE Amex Equities Sec. 713 specify similar rules for discounted private offerings involving more than 20% of existing shares.
shareholder rejects a co-financing request on a debt payment date. As a result, firm default and co-financing rejections coincide, \( \delta_t = \delta_{A,t} \).

Proof. See Appendix A.1. ■

While I provide a detailed proof of Lemma 1 in Appendix A.1, I discuss the intuition underlying this result here. Between debt payment dates, net-payout to shareholders is non-negative, implying that defaulting is never optimal. Moreover, when the large shareholder is willing to co-finance a fraction \( \omega \) of the external funds needed on a debt payment date, it must be the case that co-financing the remaining fraction \( (1 - \omega) \) is also optimal for the other investors. Since atomistic shareholders can free-ride on the large shareholder’s information without incurring information production cost, the per-share value of equity to passive investors is weakly larger than the per-share value the large shareholder assigns after accounting for information acquisition cost. Conversely, if the large shareholder is not willing to co-finance and breaks the match with the firm, then the value of the large shareholder’s \( \omega \)-stake in the equity without further information investments must be worth less than the \( \omega \)-share of the required debt payments. This, however, also implies that the total equity value is smaller than the total payment required to debt holders, rendering default optimal for equity holders.

The following lemma further greatly simplifies the characterization of equilibrium beliefs.

**Lemma 2.** It is weakly optimal for the large shareholder to truthfully reveal any information it obtains.

Proof. The large shareholder can in principle benefit from maintaining asymmetric information (by not disclosing or disclosing non-truthfully) in two types of markets: in the primary market (when the firm issues new equity) and in the secondary market (through trading with other investors). Regarding the primary market, note that the equilibrium financing and default strategy described in Lemma 1 is optimal for shareholders no matter if the large shareholder discloses additional information or not. Given that management follows this rule, the large shareholder
cannot extract additional profits in the process of new equity issuances (on debt payment dates) no matter if the large shareholder discloses its signals or not. Regarding the secondary market, the above-stated assumption of lack of noise in the trading system implies that any trades initiated by the large shareholder are interpreted as informational trades, leading the no-trade theorem to apply (Milgrom and Stokey, 1982). In sum, under the described market arrangements, the large investor cannot extract additional information rents from private information after matching with a firm. ■

Lemma 2 shows that truthful revelation of information by the large shareholder is consistent with optimality in equilibrium. While the firm’s optimal default policy (Lemma 1) does not require these additional information releases, considering the case where the large shareholder indeed reveals its information significantly streamlines the remaining analysis. As the equilibrium information sets of all agents are identical in this case, the analysis requires keeping track of only one set of beliefs on the equilibrium path. Going forward, I denote by $\pi_t$ the probability under agents’ common filtration $\mathcal{F}_t$ that the firm is in the good state, $\pi_t \equiv \Pr[\theta_t = g|\mathcal{F}_t]$. I now proceed to characterizing the evolution of these beliefs.

**Lemma 3 (Bayesian updating).** The log-odds ratio $o_t \equiv \log[\pi_t/(1 - \pi_t)]$ evolves according to the following process:

$$
d o_t = f^+(Z_t) dN^+_{x,t} + f^-(Z_t) dN^-_{x,t} + (\dot{o}(Z_t) - o_t) dN_{\theta,t} + (\infty_{\theta=g} - \infty_{\theta=b} - o_t) dN_{A,t},
$$

(10)

where I define the log-Bayes factors associated with positive and negative innovations to $x$:

$$
f^+(Z) \equiv \log \left[ \frac{\lambda^+(g, Z)}{\lambda^+(b, Z)} \right] = \log \left[ \frac{1 + \Delta_x \frac{\mu_x(g, Z)}{\sigma_x(Z)^2}}{1 + \Delta_x \frac{\mu_x(b, Z)}{\sigma_x(Z)^2}} \right],
$$

(11)

$$
f^-(Z) \equiv \log \left[ \frac{\lambda^-(g, Z)}{\lambda^-(b, Z)} \right] = \log \left[ \frac{1 - \Delta_x \frac{\mu_x(g, Z)}{\sigma_x(Z)^2}}{1 - \Delta_x \frac{\mu_x(b, Z)}{\sigma_x(Z)^2}} \right].
$$

(12)
Proof. See Appendix A.2. ■

Agents update their beliefs after observing innovations to the log interest coverage ratio \( x \), after shocks to the hidden state \( \theta \), and after the large shareholder obtains signals. Bayesian updating commands that the log-odds ratio increases by the log-Bayes factor \( f^+(Z) \) after a positive innovation to \( x \), and decreases by the log-Bayes factor \( f^-(Z) \) after a negative innovation. The lemma also reveals how these log-Bayes factors are uniquely determined by the grid increments \( \Delta_x \), the drifts \( \mu_x(\theta, Z) \), and the volatility \( \sigma_x(Z) \). As the total arrival intensity \( \lambda_x(Z) \) is independent of the hidden state \( \theta \), agents do not obtain additional information from the amount of time that passes between innovations to \( x \). Note that \( o = +\infty \) and \( o = -\infty \) are well-defined states where agents know the current value of \( \theta_t \in \{g, b\} \) with certainty.

Going forward, I use the following modeling trick to take advantage of the tractability of the proposed environment with learning and free boundary problems: suppose that in equilibrium the log-likelihood ratio \( o_t \) attains values on a discrete equidistant grid with grid increments of size \( \Delta_o > 0 \). This result is obtained by imposing that parameter choices imply that the ratios \( \frac{f^+(Z)}{\Delta_o} \) and \( \frac{f^-(Z)}{\Delta_o} \) are natural numbers. As \( \Delta_o \) can be chosen to be arbitrarily small, this condition imposes effectively no constraints on parameter choices determining earnings dynamics. Yet it insures that the domain of the log-odds ratio \( o \) is an equidistant grid with increments \( \Delta_o \).

I first analyze the optimal behavior of large shareholders after matching with a firm, that is, for \( a_t = 1 \). Conditional on being matched with a firm, the relevant state variables for a large shareholder are \( (\rho, o, Z, Y) \). Let \( \Pi_t \) denote the cumulative after-tax net payout to shareholders of the firm between dates 0 and \( t \). Absent default, net payout to shareholders over an instant \([t, t + dt)\) is given by:

\[
d\Pi_t = (1 - \tau)(X_t dt - C_t dN_{C,t}) \tag{13}
\]

The large shareholder dynamically maximizes the market value of its position by optimally
choosing information investment $I_t$ and co-financing rejections $\delta_{A,t}$. The following proposition characterizes the large shareholder’s value and optimal policy functions.

**PROPOSITION 1** (Blockholder value and policies). The market value of the large shareholder is given by

$$V(\rho_t, o_t, Z_t, X_t) = \max_{\{I_t\}_{t=0}^{t=\infty}, \{\delta_{A,t}\}_{t=0}^{t=\infty} \in \{0,1\}^2} \mathbb{E} \left[ \int_t^{\tau^*_A} \frac{m_r}{m_t} (\omega d\Pi - I_r d\tau) \right] \mathcal{F}_t, \quad (14)$$

where the time of default is $\tau_A^* \equiv \inf\{s \geq t : \delta_{A,s} dN_{C,s} = 1\}$. The Hamilton-Jacobi-Bellman (HJB) equation associated with the maximization problem (14) implies that the scaled value function $\tilde{V}(\rho, o, Z)$ solves the following set of equations for all $(\rho, o, Z) \in \Omega_\rho \times \Omega_o \times \Omega_Z$:

$$0 = \max_{\tilde{I}_t \geq 0, \delta_A \in \{0,1\}} \left\{ \omega (1 - \tau) (1 - e^{-\rho}(1 - \delta_A(\rho, o, Z))) - \tilde{I}(\rho, o, Z) \right. \right.$$  

$$- (r_f(Z) + r_{p_A}(\rho, o, Z) - \mu_Y(Z) + \lambda_C \delta_A(\rho, o, Z)) \cdot \tilde{V}(\rho, o, Z) \right.$$  

$$+ \psi \tilde{I}(\rho, o, Z)^n \cdot (\pi(o) \cdot \tilde{V}(\rho, \infty, Z) + (1 - \pi(o)) \cdot \tilde{V}(\rho, -\infty, Z) - \tilde{V}(\rho, o, Z)) \right.$$  

$$+ \Lambda_{\rho, o}(\rho, o, Z) \tilde{V}_{\rho, o}(\rho, o, Z) + \Lambda_Z(Z) \tilde{V}_Z(\rho, o, Z) \right\}, \quad (15)$$

where $I$ define $\pi(o) \equiv \frac{e^o}{1 + e^o}$, where $\tilde{V}_{\rho, o}(\rho, o, Z)$ indicates a vector that collects the values of the function $\tilde{V}(\rho, o, Z)$ evaluated at all $(\rho, o) \in \Omega_\rho \times \Omega_o$ while keeping the other arguments fixed, and where the matrix $\Lambda_{\rho, o}(\rho, o, Z)$ reflects that the states $(\rho, o)$ move simultaneously respecting the evolution of the log-odds ratio $o$ stated in equation (3). The expression for a large shareholder’s risk premium $r_{p_A}$ is given in Appendix A.3. The optimal controls solving (15) are given by:

$$\tilde{I}(\rho, o, Z) = \max \left\{ \psi \eta \pi(o) \cdot \tilde{V}(\rho, \infty, Z) + (1 - \pi(o)) \cdot \tilde{V}(\rho, -\infty, Z) - \tilde{V}(\rho, o, Z), 0 \right\} \frac{1}{1 - \eta}, \quad (16)$$

$$\delta_A(\rho, o, Z) = 1_{\{\tilde{V}(\rho, o, Z) < \omega e^{c(1 - \tau)}\}}. \quad (17)$$

**Proof.** See Appendix A.3.  

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The tractability of the proposed setup follows from the fact that conditional on any policy functions $\tilde{I}(\rho, o, Z)$ and $\delta_A(\rho, o, Z)$, the value function $\tilde{V}(\rho, o, Z)$ is available in closed form — equation (15) represents a linear system that can be solved by inverting a sparse matrix. As a result, obtaining precise solutions to the free-boundary problem with endogenous information acquisition is straightforward and fast.

Next, I turn to characterizing the value of the firm’s equity. The relevant state vector for the equity value is $(a, \rho, o, Z, X)$.

**PROPOSITION 2** (Equity value). The total equity market value is given by:

$$P(a_t, \rho_t, o_t, Z_t, X_t) = \max_{\{\delta_r\}^\infty}_{\tau_m \in \{0,1\}} \mathbb{E} \left[ \int_t^{\tau^*} \frac{m_r}{m_t} d\Pi_t \left| \mathcal{F}_t \right. \right],$$

where $\tau^* \equiv \inf\{s \geq t : \delta_s dN_{C,s} = 1\}$. The HJB equation associated with the maximization problem in (18) implies that the scaled value function $\tilde{P}(a, \rho, o, Z)$ solves the following set of equations for all $(a, \rho, o, Z) \in \Omega_a \times \Omega_\rho \times \Omega_o \times \Omega_Z$:

$$0 = \{(1 - \tau)(1 - e^{-\rho}(1 - \delta(a, \rho, o, Z)))$$

$$- (r_f(Z) + rp(a, \rho, o, Z) - \mu_Y(Z) + \lambda_C \delta(a, \rho, o, Z)) \cdot \tilde{P}(a, \rho, o, Z)$$

$$+ a\psi \tilde{I}(\rho, o, Z)^r \cdot (\pi(o) \cdot \tilde{P}(a, \rho, -\infty, Z) + (1 - \pi(o))\tilde{P}(a, \rho, -\infty, Z) - \tilde{P}(a, \rho, o, Z))$$

$$+ \Lambda_{\rho,o}(a, \rho, o, Z)\tilde{P}_{\rho,o}(a, \rho, o, Z) + \Lambda_Z(Z)\tilde{P}_Z(a, \rho, o, Z)\}.$$

The expression for a firm’s equity risk premium $rp_P$ is given in Appendix A.4. The optimal controls are given by:

$$\delta(1, \rho, o, Z) = \delta_A(\rho, o, Z),$$

$$\delta(0, \rho, o, Z) = \mathbb{1}_{\{\tilde{P}(0,\rho,o,Z) < e^{(1-\tau)}\}}.$$

Lemma 1 implies that conditional on a match, $(a = 1)$, the firm optimally defaults when
the large shareholder rejects to co-finance. Otherwise, for \( a = 0 \), the firm defaults when the equity continuation value is below the required payment to debt holders. Equation (37) does not explicitly reflect the arrival rate with which a firm matches with a large shareholder (that is, \( a \) switching from 0 to 1), since these shocks are value-neutral to existing equity holders — given the large investor’s take-it-or-leave-it offer, the large investor extracts the incremental equity value created by her future efforts. Management has the outside option to reject and wait for the next large investor, but the next investor will again make a take-it-or-leave it offer. Once a match is established, there is no incentive to break the match unless there is also a firm default. As a result, the systems of equations determining the values \( \bar{P}(1, \rho, o, Z) \) and \( \bar{P}(0, \rho, o, Z) \) can be solved separately. Conditional on the policy functions, value functions are again available in closed-form due to the linearity of the system (37) in \( \bar{P}(a, \rho, o, Z) \).

The following lemma characterizes large investors’ decisions on when to establish a match with a firm.

**Lemma 4** (Matching between large investors and firms). *Conditional on obtaining a negotiation opportunity with a firm, a large investor implements a transaction if and only if the value created exceeds the fixed negotiation costs:*

\[
\frac{\tilde{V}(\rho, o, Z) + (1 - \omega)\bar{P}(1, \rho, o, Z)}{\text{Total value with large shareholder}} - \frac{\bar{P}(0, \rho, o, Z)}{\text{Total value without large shareholder}} > \bar{\chi}. \tag{22}
\]

*The ex ante value of surplus extracted by the group of large investors with a given firm is characterized in Appendix A.6.*

Given these endogenous matching decisions, we can now proceed to characterizing the value of debt in all states.
PROPOSITION 3 (Debt value).

\[
D(a_t, \rho_t, \alpha_t, Z_t, X_t) = \mathbb{E} \left[ \int_t^\tau \frac{m_t}{m_t} C_t dN_{C_t} \bigg| \mathcal{F}_t \right]
\]

(23)

The HJB equation associated with (23) implies that the scaled debt value \( \tilde{D}(a, \rho, o, Z) \) solves the following set of equations for all \( (a, \rho, o, Z) \in \Omega_a \times \Omega_x \times \Omega_o \times \Omega_Z \):

\[
0 = \left\{ e^{-\rho}(1 - \delta(a, \rho, o, Z)) - (\gamma_1(Z) + \rho p_D(a, \rho, o, Z) - \mu_Y(Z)) \cdot \tilde{D}(a, \rho, o, Z)
\right.
\]

\[
+ \lambda_C \delta(a, \rho, o, Z) \cdot (\alpha(Z) \cdot \tilde{U}(a, Z) - \tilde{D}(a, \rho, o, Z))
\]

\[
+ \lambda_C \cdot (1 - a) \cdot \kappa \cdot \mathbb{E} \left[ \tilde{V}(\rho, o, Z) + (1 - \omega) \tilde{p}(1, \rho, o, Z) - \tilde{p}(0, \rho, o, Z) \right] \cdot (\tilde{D}(1, \rho, o, Z) - \tilde{D}(0, \rho, o, Z))
\]

\[
+ a \psi \tilde{l}(\rho, o, Z)^\rho \cdot (\pi(o) \cdot \tilde{D}(1, \rho, -\infty, Z) + (1 - \pi(o)) \cdot \tilde{D}(1, \rho, -\infty, Z) - \tilde{D}(1, \rho, o, Z))
\]

\[
+ \lambda_Z(\rho, o, Z) \tilde{D}_z(\rho, o, a, o, Z) + \Lambda_Z(Z) \tilde{D}_Z(\rho, o, a, o, Z) \right\},
\]

(24)

where the expression for the debt risk premium \( \rho p_D \) is given in Appendix A.4, and where \( U \) denotes the value of the unlevered firm, which is characterized in Appendix A.5.

In states where a firm is not yet matched with a large shareholder \( (a = 0) \), the debt value encodes a potential future match \( (a \rightarrow 1) \), which affects default policies and thus also debt holders’ value. Moreover, equation (24) reflects that in default, debt holders recover a fraction \( \alpha(Z) \) of the firm’s unlevered assets.

4. Calibration and Evaluation

In this section, I calibrate the model and evaluate its predictions.
4.1. Choosing Parameters

In the following, I discuss the parameter choices determining the dynamics of exogenous macroeconomic processes, and the technologies of firms and large investors. Parameter values are listed in Table 1. I use values from the existing literature to calibrate the dynamics of the macroeconomic processes \( Z \) and \( Y \) and of the stochastic discount factor (Chen, Xu, and Yang, 2013, Binsbergen and Opp, 2018). This calibration considers two aggregate states \( Z \in \{\text{boom, recession}\} \). Transition rates between these aggregate states are set such that expansions and recessions last, on average, 10 years and 2 years, respectively.

Conditional on the dynamics of the state variables \( Z \) and \( Y \), a firm’s earnings dynamics are fully determined by the drifts \( \mu_z(\theta, Z) \) and the volatilities \( \sigma_z(Z) \). I choose the expected earnings growth rate differentials across aggregate states to match those estimated in Bhamra, Kuehn, and Strebulaev (2010). Moreover, the differences in growth rates across firms in states \( g \) and \( b \) are chosen to ensure that under the stationary distribution, the mapping between firm default rates and interest coverage ratios is consistent with the data (Moody’s, 2017, 2018). These earnings dynamics ensure that firms earn an equity risk premium of about 6% at a leverage of 30%. Overall earnings volatility is specified to match estimates in Bhamra, Kuehn, and Strebulaev (2010).\(^{11}\) Similarly, recovery rates in default, \( \alpha(Z) \), are taken from Bhamra, Kuehn, and Strebulaev (2010).

Given the fact that the empirical distribution of equity stakes acquired in PIPE transactions of distressed firms exhibits significant bunching at the regulatory 20% threshold (Park, 2011), I consider the acquisitions stake \( \omega = 0.2 \) as a benchmark value. I set the parameters \( \kappa \) and \( \chi \) such that under the stationary distribution, matching with large investors occurs when a firm’s interest coverage ratio is typically between 0.5 and 0.7, corresponding to the average and median interest coverage ratio of firms rated Caa-C (Moody’s, 2017). These interest coverage values

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\(^{11}\) In Bhamra, Kuehn, and Strebulaev (2010) local idiosyncratic volatility is 22.6 percent and local systematic volatility is on average 10.1 percent.
Table 1

Parameters. The three panels list parameters of the macroeconomy, firms, and large investors. The grid increments for the log interest coverage ratio and the log-odds ratio are given by $\Delta_x = 0.095$ and $\Delta_o = 0.018$, respectively. The parameter choices imply that the log-Bayes factor associated with a positive earnings innovation, $f^+(Z)$, is $8\Delta_o$ in booms and $9\Delta_o$ in recessions. Similarly, the log-Bayes factor for negative innovations, $f^-(Z)$, is $9\Delta_o$ in booms and $8\Delta_o$ in recessions. The arrival intensity of interest payments implies that payments occur on average every quarter.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Boom</th>
<th>Recession</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical transition rates for aggregate states</td>
<td>$\lambda_Z$</td>
<td>0.100</td>
<td>0.500</td>
</tr>
<tr>
<td>Risk neutral transition rates for aggregate states</td>
<td>$\tilde{\lambda}_Z$</td>
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<td>0.250</td>
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<tr>
<td>Trend growth</td>
<td>$\mu_Y$</td>
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<td>−0.015</td>
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<tr>
<td>Trend volatility</td>
<td>$\sigma_Y$</td>
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<td>0.029</td>
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<tr>
<td>Risk free rate</td>
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<td>0.050</td>
</tr>
<tr>
<td>Local risk price</td>
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<td>0.700</td>
<td>1.300</td>
</tr>
<tr>
<td>Tax rate</td>
<td>$\tau$</td>
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</tr>
</tbody>
</table>

Firms

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Boom</th>
<th>Recession</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drift of $x$ in good firm state ($\theta = g$)</td>
<td>$\mu_x$</td>
<td>0.080</td>
<td>0.010</td>
</tr>
<tr>
<td>Drift of $x$ bad firm state ($\theta = b$)</td>
<td>$\mu_x$</td>
<td>−0.080</td>
<td>−0.010</td>
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<tr>
<td>Volatility of $x$</td>
<td>$\sigma_x$</td>
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<td>0.240</td>
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<tr>
<td>Arrival rate of shocks to hidden state</td>
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<td></td>
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<tr>
<td>Arrival rate of interest payments</td>
<td>$\lambda_C$</td>
<td>4.000</td>
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</table>

<table>
<thead>
<tr>
<th>Large Investors</th>
<th>Variable</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of matching opportunity on payment dates</td>
<td>$\kappa$</td>
<td>0.500</td>
</tr>
<tr>
<td>Fixed costs of negotiation</td>
<td>$\tilde{\chi}$</td>
<td>0.300</td>
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<tr>
<td>Information investment efficiency</td>
<td>$\psi$</td>
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<tr>
<td>Decreasing returns to scale parameter</td>
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</tr>
<tr>
<td>Ownership stake</td>
<td>$\omega$</td>
<td>0.200</td>
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</table>

are also representative of firms categorized as financially distressed in Andrade and Kaplan (1998). I choose a decreasing returns to scale parameter of 0.6, similar to standard assumptions on decreasing returns at the firm level. I set the information investment efficiency parameter $\psi$ to match a typical gain of about 20% for large investors from PIPE transactions, consistent with empirical estimates (Park, 2011).
4.2. Results of the Calibration

In this section, I analyze the predictions of the calibrated model. Throughout, the presented figures plot outcomes as a function of the state variables. Given the high dimensionality of the state space, the graphs focus on three belief levels \( \pi \in \{0.07, 0.22, 0.39\} \) that correspond to the 20th, 50th, and 80th percentile under the stationary distribution, conditional on having an interest coverage ratio below 1. The reason for these relatively pessimistic beliefs is that earnings innovations and beliefs are correlated. When moving from an interest coverage ratio of around 10, which is representative for A-rated firms (Moody’s, 2017), to an interest coverage ratio below 1, typical for C-rated firms, firms have experienced a series of negative earnings innovations that have caused agents to update their beliefs downwards. The horizontal axes of the figures represent the interest coverage ratio \( e^\theta \). Finally, the figures consistently feature two panels that separately illustrate outcomes in booms and recessions.

**Information acquisition.** Figure I illustrates the endogenous arrival rate of information obtained by a large shareholder conditional on having matched with a firm. The figure reveals the highly non-linear behavior of large shareholders’ efforts. For sufficiently high earnings, the firm is very likely to be solvent and deserving of continued support from shareholders, implying that incentives for incremental information acquisition are low. Yet, for lower interest coverage ratios, attention to the firm’s conditions increases strongly, peaking at an interest coverage ratio that implies high one-year default rates (see Panels (a) and (b) of Figure II). If interest coverage declines even further, incentives for information acquisition decline again as the chances of firm solvency become minimal. In sum, when the firm is very likely to be either solvent or insolvent, incentives for further information acquisition are low. In contrast, when there is uncertainty about firm solvency and shareholders’ benefits from providing continued support to the firm, the large investor pays significantly more attention to evaluating the firm’s prospects.
**FIGURE I**

**Information acquisition and leverage.** Panels (a) and (b) plot the arrival intensity of signals obtained by the large shareholder in booms and in recessions, respectively. Panels (c) and (d) illustrate market leverage in booms and recessions, respectively. The graphs consider three belief levels that are representative of the 20th, 50th, and 80th percentile of beliefs under the stationary distribution. The horizontal axis represents the firm’s interest coverage ratio \(\hat{X}\). All parameter values are provided in Table 1.

**Default rates.** Panels (a) and (b) of Figure II illustrate the one-year default rates of a firm that has a large shareholder. The Panels show that apart from the interest coverage ratio, beliefs and the state of the business cycle play an important role in determining default risk. Moreover, Panels (c) and (d) reveal the non-monotone impact of the large shareholder’s information acquisition on default rates. These Panels compare the state-contingent one-year default rates.
FIGURE II

**One-year default rates and the impact of large investors.** The figure illustrates the conditional one-year default rates (Panels (a) and (b)) and the change in these default rates relative to those obtaining in a counterfactual economy without large investors (Panels (c) and (d)). The panels plot these measures in booms (Panels (a) and (c)) and in recessions (Panels (b) and (d)) as a function of the interest coverage ratio $\bar{\lambda}$, and for three distinct belief levels. The three illustrated belief levels correspond to the 20th, 50th, and 80th percentile for beliefs under the stationary distribution conditional on an interest coverage ratio below 1. All parameter values are provided in Table 1.

with large shareholder involvement to those in a counterfactual economy without large shareholders. Conditional default rates are substantially increased for low and intermediate levels of distress, but they are decreased very close to default. For low and intermediate levels of distress, the large shareholder frequently obtains information confirming the firm’s insolvency, causing
shareholders to withdraw their support more quickly. In contrast, for extremely distressed firms with uncertain prospects, the involvement of a large shareholder tends to prolong the firm’s life expectancy — as shareholders anticipate to learn more about the firm’s true condition from the large shareholder in the near future, it is optimal for them to continue supporting the firm until they have more definitive information about the firm’s prospects. This non-monotone impact of large investors on default risk is of first-order importance for externalities on equity, debt, and firm value, which I discuss in more detail next.

**Impact on equity values.** Figure III illustrates large investors’ impact on state-contingent equity and debt values for a matched firm \((a = 1)\) relative to a counterfactual economy without large shareholders. All value changes are scaled by the state-contingent firm values in the counterfactual economy. Panels (a) and (b) illustrate the impact on equity values, showing a hump-shaped pattern as a function of the interest coverage ratio, which is qualitatively similar to the pattern obtained for the large shareholder’s information acquisition (Figure I). Comparing equity value gains across different belief levels confirms the intuition that gains are limited if agents are already quite certain that the firm is in a bad state (at the 20th percentile of beliefs the chance of being in a bad state is 93%).

It is worth highlighting that the illustrated ratios of gains are affected by both the equity gains (the numerator) and the “autarky” firm value (the denominator). The autarky firm value declines with lower interest coverage ratios not only because of lower earnings, but also because of a higher chance of default and associated distress costs (which include tax shield losses given the standard assumption that debt holders recover a fraction \(\alpha(Z)\) of the unlevered firm value).

When interpreting the illustrated value gains, it is important to keep in mind that they differ fundamentally from “announcement returns” that obtain for existing equity holders conditional on observing a new match between a firm and a large investor. These announcement returns are in fact negative for two reasons. First, equity value gains from information acquisition are
(a) Equity value gain (Boom)

(b) Equity value gain (Recession)

(c) Debt value gain (Boom)

(d) Debt value gain (Recession)

**FIGURE III**

**Impact on equity and debt values.** The figure illustrates the gain in equity and debt values for a firm that has a large shareholder, relative to a counterfactual economy without large shareholders. The panels plot these value gains for equity and debt in booms (Panels (a) and (c)) and in recessions (Panel (b) and (d)) as a function of the interest coverage ratio $\bar{X}$, and for three distinct belief levels. The three illustrated belief levels correspond to the 20th, 50th, and 80th percentile for beliefs under the stationary distribution conditional on an interest coverage ratio below 1. All parameter values are provided in Table 1.

extracted by large investors through negotiated PIPE transactions in which existing atomistic shareholders are pushed to their outside option. Second, transactions coincide with payment shocks $dN_C = 1$ that are negative news for equity values. Negative returns on announcement dates can therefore not be taken as evidence that large shareholders destroy equity value or that
existing equity holders are worse off when a large investor acquires an equity stake.

**Impact on debt values.** Large shareholders’ impact on debt values is a priori theoretically ambiguous. On the one hand, large shareholders’ information facilitates the optimal exercising of equity holders’ put option, in which debt investors hold a short position. That is, large shareholders’ increased attention to firm conditions generates a redistribution of value from debt to equity holders. On the other hand, the total value to be distributed between debt and equity holders — that is, firm value — may increase as the put value gain for equity holders lowers the incidence of default in any given state of the world. Yet, as large shareholders’ information acquisition also affects the probability distribution of states, it generally has an ambiguous effect on default probabilities, as demonstrated by the above-discussed one-year default rates (Figure II). The possibility of lower default risk, in turn, implies the possibility of gains for debt holders.

Reflecting these competing effects, Panels (c) and (d) of Figure III reveal distinct patterns for debt value changes in booms and recessions. In booms, debt value changes exhibit an inverse hump-shaped pattern as a function of the interest coverage ratio. In these states, debt values are consistently negatively affected by the actions of the large shareholder. In contrast, in recessions, debt values are positively affected for highly distressed firms. As highlighted above, the large shareholder does tend to reduce default risk for highly distressed firms with uncertain growth prospects. Moreover, since default leads to greater deadweight losses in recessions (recovery rates are lower), these reductions in default risk have a larger positive effect on overall firm value. In net, a large shareholder can therefore positively affect debt values in highly distressed states, in particular when the deadweight losses from bankruptcy are material.

Again, these illustrated value changes differ fundamentally from announcement returns. Prior to matching with a large shareholder, debt market prices already encode the possibility of future investor involvement (a switch from \( a = 0 \) to \( a = 1 \)). When the matching with a
large investor becomes highly likely in the near-term, debt trades already close to the value that obtains once a PIPE transaction has occurred. Thus, announcement returns are generally a poor measure of the impact of large shareholders on debt values.

**Impact on overall firm value.** Panels (a) and (b) of Figure IV combine the discussed value changes for equity and debt to evaluate the implications for overall firm value. The graphs again reveal a non-monotone pattern, consistent with the above-discussed implications for default risk illustrated in Figure II. Large shareholders affect firm value through their impact on the efficiency of default decisions. In recessions, when default risk leads to greater inefficiencies, any changes in default rates have a more material effect on overall firm value. As a result, firm value gains can be quite substantial for highly distressed firms in recessions. In contrast, in booms, the effects are dampened by smaller distress costs. In this context it is useful to highlight that in the limit, when default does not lead to any inefficiencies (for \( \tau = 0 \) and \( \alpha(Z) = 0 \)), the value implications of large shareholders for overall firm value are nil — in this case, large shareholders only cause a redistribution of value from debt to equity claims.\(^{12}\)

**Endogenous timing of PIPE transactions.** Figure V illustrates the probabilities with which a large investor will match with the firm the next time the firm needs to raise external funds. The graphs reveal that PIPE transactions are more likely to occur when there is substantial uncertainty about the firm’s prospects, which is the case at the illustrated 50th and 80th percentiles of beliefs. In contrast, at the 20th percentile of beliefs, agents know with 93% probability that the firm is in the bad state, implying that PIPE transactions are not negotiated unless the firm is very close to default. Interestingly, matching and negotiation frictions causing such delays in large shareholder involvement can have positive implications for overall firm value. As highlighted in the previous discussion of Figure IV, the efficiency-enhancing effects of large shareholders’ ac-

\(^{12}\)Taxes affect losses conditional on default as debt holders are assumed to collect a fraction \( \alpha(Z) \) of the unlevered firm value, implying that losses are due to both bankruptcy costs and reduced future interest tax shields.
FIGURE IV

Impact on firm value. The figure illustrates the gain in firm value for a firm that has a large shareholder, relative to a counterfactual economy without large shareholders. The panels plot these value gains in booms (Panel (a)) and in recessions (Panel (b)) as a function of the interest coverage ratio $\bar{X}$, and for three distinct belief levels. The three illustrated belief levels correspond to the 20th, 50th, and 80th percentile for beliefs under the stationary distribution conditional on an interest coverage ratio below 1. All parameter values are provided in Table 1.

Interventions dominate the value-destroying rent-seeking effects for "last minute" interventions, that is, interventions in states in which the firm would be very likely to go bankrupt on the next payment date unless a PIPE transaction occurs. These results suggest that frictions causing PIPE transactions to predominantly occur in the form of such last-minute interventions can have positive implications for overall efficiency.

5. Conclusion

Large shareholdings are a wide-spread empirical phenomenon that can have first-order effects on investors’ efforts to influence the decisions of firms and other investors. In this paper, I analyze the externalities posed by large shareholders in the context of financial distress where the quality of investors’ information about firm solvency is essential for firm survival. I pro-
FIGURE V
Probability of matching when next debt payment is due. The figure illustrates the probability with which a large investor matches with an unmatched firm \((\alpha = 0)\) when the next debt payment is due. The two panels plot these probabilities in booms (Panel (a)) and in recessions (Panel (b)) as a function of the interest coverage ratio \(\tilde{X}\), and for three distinct belief levels. The three illustrated belief levels correspond to the 20th, 50th, and 80th percentile for beliefs under the stationary distribution conditional on an interest coverage ratio below 1. All parameter values are provided in Table 1.

Pose a highly tractable dynamic model of the interplay between large shareholders and indebted firms’ atomistic equity holders that nests central features of standard structural credit risk models. The model yields precise global solutions that reveal the highly nonlinear behavior of large shareholders’ attention and its associated externalities. Information acquisition on the firm’s prospects causes large shareholders to take a pivotal role in distressed firm’s financing, materially affecting not only the distribution of value across different claims but also overall efficiency. Large shareholders’ impact on firm value is highly non-monotone in financial distress, turning positive only in deeply distressed states. The model sheds light on the implications of equity issuances via private investments in public equity (PIPE transactions) and frictions making such transactions more likely to occur as “last minute” interventions that support deeply distressed firms.
A. Proofs

A.1. Proof of Lemma 1

Between payment dates, net-payout to shareholders is non-negative implying that shutting down the firm is never optimal in those times. Suppose that management indeed follows the decision rule to default if and only if the large shareholder refuses to co-finance on a payment date. Given this decision rule, the large shareholder knows that it effectively controls default decisions with its accept/reject response. Thus, the large shareholder maximizes its value by choosing the information investment rate $I$ and whether to effectively trigger default $\delta_A \in \{0, 1\}$ conditional on the arrival of a payment date:

$$
\max_{\{I_t\}_{\tau=t}^{\infty}, \{\delta_{A,\tau}\}_{\tau=t}^{\infty} \in \{0, 1\}} \mathbb{E} \left[ \int_t^{\tau^*} \frac{m_r}{m_t} (\omega d\Pi_{\tau} - I_{\tau} d\tau) \middle| \mathcal{F}_{A,t} \right],
$$

(25)

where $\mathcal{F}_{A,t}$ denotes the large shareholder’s filtration, and where the time of default is $\tau^* \equiv \inf\{s \geq t : \delta_{A,s} dN_{C,s} = 1\}$. Note that given the offers from management after the initial transaction, the large shareholder by construction consistently collects a fraction $\omega$ of net-payout up until it refuses to co-finance and triggers default. Next, we need to verify whether management indeed optimally chooses the conjecture decision rule if the large shareholder chooses $\{I_t\}_{\tau=t}^{\infty} \geq 0, \{\delta_{A,\tau}\}_{\tau=t}^{\infty} \in \{0, 1\}$ to maximize (25). First, if the large shareholder accepts to co-finance a required debt payment at date $t$, then it must be the case that the maximized objective (25) is weakly larger than $\omega C_t$. If (25) is weakly larger than $\omega C_t$, then it must also be the case that:

$$
\mathbb{E} \left[ \int_t^{\tau^*} \frac{m_r}{m_t} (1 - \omega) d\Pi_{\tau} \middle| \mathcal{F}_{A,t} \right] > (1 - \omega)C_t,
$$

(26)
since (25) also includes the investment cost \( I \geq 0 \) (which lowers the objective). Given the inequality (26), management indeed optimally keeps the firm afloat when the large shareholder co-finances (notice that (25) conditions on the filtration of the large shareholder \( \mathcal{F}_{A,t} \), which is weakly larger than other agents’ filtration). Second, if the large shareholder does not accept to co-finance at a payment date \( t \) then it must be the case that (25) is smaller than \( \omega C_t \). Notice that by optimality of the choice of \( \{I_r\}_{r=t}^\infty \geq 0 \) it must be the case that (25) is weakly greater than the following objective that restricts the large shareholder to set \( \{I_r\}_{r=t}^\infty = 0 \):

\[
\max_{\{\delta_{A,r}\}_{r=t}^\infty \in \{0,1\}} \mathbb{E} \left[ \int_t^{\tau^{**}} \frac{m_r}{m_t} \omega d\Pi_r \bigg| \mathcal{F}_{A,t} \right],
\]

where the time of default \( \tau^{**} \equiv \inf\{s \geq t : \delta_{A,s}d\mathcal{N}_{C,s} = 1\} \) is now chosen optimally without access to additional signals since \( \{I_r\}_{r=t}^\infty = 0 \). Thus, if (25) is smaller than \( \omega C_t \) then it must also be the case that the constrained maximum (27) is smaller than \( \omega C_t \). Dividing (27) by \( \omega \) and dividing \( \omega C_t \) by \( \omega \) then also implies that the following inequality holds if the large shareholder refuses to co-finance:

\[
\max_{\{\delta_{A,r}\}_{r=t}^\infty \in \{0,1\}} \mathbb{E} \left[ \int_t^{\tau^{**}} \frac{m_r}{m_t} d\Pi_r \bigg| \mathcal{F}_{A,t} \right] < C_t.
\]

Inequality (28) implies that even if the shareholders other than the large shareholder received all dividends going forward, the shareholders would optimally reject to make the payment \( C_t \) (under the richer information set of the large shareholder \( \mathcal{F}_{A,t} \)). This result takes advantage of another fact: as large investors make take-it-or-leave-it-offers to firms, the equity absent large shareholder involvement is priced exactly as if the probability of future involvement was zero (in any state without involvement, \( a = 0 \)), implying that the left-hand side of (28) indeed represents the continuation value for shareholders if the large shareholder rejects to co-finance. As a result, management will optimally trigger default when the large shareholder refuses to co-finance. Finally, note that in unambiguously solvent states (when \( P > C \) independent of beliefs
about \( \theta \), the large shareholder could reject co-financing without causing firm default. Yet doing so is suboptimal since it breaks the match with the firm and implies that the large shareholder can no longer increase the value of its claim by acquiring information going forward.

A.2. Proof of Lemma 3

In the following, I derive the representations for the log-Bayes factors (12) and (11). By definition Bayes factors associated with positive and negative innovations are given by:

\[
e^{f^+(Z)} = \frac{\lambda^+_x(g, Z)}{\lambda^+_x(b, Z)},
\]

\[
e^{f^-(Z)} = \frac{\lambda^-_x(g, Z)}{\lambda^-_x(b, Z)}.
\]

In addition, restriction (6) implies the following equations:

\[
\lambda_x(Z) = \lambda^+_x(g, Z) + \lambda^-_x(g, Z),
\]

\[
\lambda_x(Z) = \lambda^+_x(b, Z) + \lambda^-_x(b, Z).
\]

Combining these equations with the formulas for \( \mu_x(\theta, Z) \) and \( \sigma_x(Z) \) provided in equations (7) and (8) yields the provided formulas for the log-Bayes factors.

A.3. Proof of Proposition 1

Lemma 1 implies that in equilibrium, the large shareholder optimally rejects co-financing only when firm default is also optimal for all equity holders, that is, optimal co-financing strategies reject only in states where these rejections also trigger firm default. Thus, for a firm that is matched with a large shareholder, the time of default is simply given by: \( \tau^*_A \equiv \inf\{s \geq t : \delta_A(s) dN_{C,s} = 1\} \). The Hamilton-Jacobi-Bellman equation associated with the maximization
problem in (14) is given by:

\[
0 = \max_{T \geq 0, \delta_A \in \{0, 1\}} \left\{ \omega(1 - \tau)(Y e^\tau - \lambda_C(1 - \delta_A(\rho, o, Z))Ye^\tau) - Y \tilde{I}(\rho, o, Z) \right. \\
- (r_f(Z) + \lambda_C\delta_A(\rho, o, Z)) \cdot V(\rho, o, Z, Y) \\
+ V_Y(\rho, o, Z, Y)Y \mu_Y(Z) + \frac{1}{2} V_{YY}(\rho, o, Z, Y)Y^2 \sigma_Y(Z)^2 - V_T(\rho, o, Z, Y)Y \sigma_Y(Z)\nu(Z) \\
+ \psi I(\rho, o, Z)^n \cdot (\pi(o) \cdot V(\rho, \infty, Z, Y) + (1 - \pi(o)) \cdot V(\rho, -\infty, Z, Y) - V(\rho, o, Z, Y)) \\
\left. + \Lambda_{\rho, o}(\rho, o, Z)V_{\rho, o}(\rho, o, Z, Y) + \Lambda_Z(Z) V_Z(\rho, o, Z, Y) \right\},
\]

where I define \( \pi(o) \equiv \frac{e^p}{1+e^p} \), where \( V_{\rho, o}(\rho, o, Z, Y) \) indicates a vector that collects the values of the function \( \tilde{V}(\rho, o, Z) \) evaluated at all \( (\rho, o) \in \Omega_\rho \times \Omega_o \) while keeping the other arguments fixed, and where the matrix \( \Lambda_{\rho, o}(\rho, o, Z) \) reflects that the states \( (\rho, o) \) move simultaneously respecting the evolution of the log-odds ratio \( o \) stated in equation (3). Given the conjecture that \( V(\rho_t, o_t, Z_t, Y_t) = Y \cdot \tilde{V}(\rho, o, Z) \), it can be verified that the HJB equation scales with \( Y \). Dividing by \( Y \), rearranging and using the risk premium definition

\[
rp_A(\rho, o, Z) = \sigma_Y(Z)\nu(Z) + (\Lambda_Z(Z) - \Lambda_Z(Z)) \frac{\tilde{V}_Z(\rho, o, Z)}{\tilde{V}(\rho, o, Z)}.
\]

yields equation (15).

A.4. Proposition 2

The derivation of equation (37) takes advantage of the scaling property that was shown to hold in the proof of Proposition 1 in Appendix A.3. The equity risk premium \( rp_P \) is given by:

\[
rp_P(a, \rho, o, Z) = \sigma_Y(Z)\nu(Z) + (\Lambda_Z(Z) - \Lambda_Z(Z)) \frac{\tilde{P}_Z(a, \rho, o, Z)}{\tilde{P}(a, \rho, o, Z)}.
\]
A.5. Unlevered Equity Value

The unlevered firm does not face a default decision and thus, its value is not affected by the presence of a large shareholder. The unlevered equity market value is given by:

$$U(o_t, Z_t, X_t) = \mathbb{E} \left[ \int_t^{\tau^*} \frac{m_t}{m_t} (1 - \tau) X_t d\tau \middle| \mathcal{F}_t \right],$$  \hspace{1cm} (36)

where the scaled unlevered value $\tilde{U}(o, Z)$ solves the following set of equations for all $(o, Z) \in \Omega_o \times \Omega_Z$:

$$0 = \left\{ \begin{array}{l} (1 - \tau) - (r_f(Z) + rp_U(o, Z) - \mu_Y(Z)) \cdot \tilde{U}(o, Z) \\
+ \Lambda_o(o, Z) \tilde{U}_o(o, Z) + \Lambda_Z(Z) \tilde{U}_Z(o, Z) \end{array} \right\},$$  \hspace{1cm} (37)

and where the risk premium is given by:

$$rp_U(o, Z) = \sigma_Y(Z) \nu(Z) + (\Lambda_Z(Z) - \bar{\Lambda}_Z(Z)) \frac{\tilde{U}_Z(o, Z)}{U(o, Z)}. \hspace{1cm} (38)$$

A.6. Value Extracted by Large Investors

Let $W$ denote the ex ante value of the surplus extracted by the group of large investors with a given firm before a match has occurred ($a = 0$). The scaled value $\tilde{W}$ solves the following set of equations for $(\rho, o, Z) \in \Omega_\rho \times \Omega_o \times \Omega_Z$:

$$0 = \left\{ \begin{array}{l} \lambda_C \cdot \kappa \cdot \max \left[ \tilde{V}(\rho, o, Z) + (1 - \omega) \tilde{P}(1, \rho, o, Z) - \tilde{P}(0, \rho, o, Z) > \bar{x}, 0 \right] \\
- \lambda_C \cdot (1 - a) \cdot \kappa \cdot \mathbb{1}_{\{\tilde{V}(\rho, o, Z) + (1 - \omega) \tilde{P}(1, \rho, o, Z) - \tilde{P}(0, \rho, o, Z) > \bar{x} \}} \tilde{W}(\rho, o, Z) \\
- (r_f(Z) + rp_W(\rho, o, Z) - \mu_Y(Z)) \cdot \tilde{W}(\rho, o, Z) \\
+ \Lambda_{\rho, o}(\rho, o, Z) \tilde{W}_{\rho, o}(\rho, o, Z) + \Lambda_Z(Z) \tilde{W}_Z(\rho, o, Z) \end{array} \right\},$$  \hspace{1cm} (39)
where I define the risk premium:

$$rp_W(\rho, o, Z) = \sigma_Y(Z)\nu(Z) + (\Lambda_Z(Z) - \bar{\Lambda}_Z(Z))\frac{\bar{W}_Z(\rho, o, Z)}{W(\rho, o, Z)}. \quad (40)$$

\section*{B. Optimal Leverage}

In this section, I characterize firms’ optimal initial leverage choices. Suppose the firm is unlevered at date $t = 0$ and its log earnings are $x_0 \in \Omega_x$. The firm chooses an interest coverage ratio $\rho_0$ from the set:

$$\Omega_\rho \in \{x - \log[\lambda_C] : x \in \Omega_x\}. \quad (41)$$

A choice $\rho_0 \in \Omega_\rho$ together with the initial log earnings level $x_0$ pins down the following choice for the log of detrended debt payments:

$$c = x_0 - \log[\lambda_C] - \rho_0. \quad (42)$$

At date $t = 0$, the firm then chooses $\rho_0$ to solve the following program:

$$\max_{\rho_0 \in \Omega_\rho} \left\{ \tilde{P}(a_0, \rho_0, o_0, Z_0) + \tilde{D}(a_0, \rho_0, o_0, Z_0) \right\}. \quad (43)$$

where the functions $\tilde{P}(a, \rho, o, Z)$ and $\tilde{D}(a, \rho, o, Z)$ are characterized Propositions 1 and 3.

\section*{References}


