Abstract

The last two decades has seen a sharp decline in the number of U.S.public firms. This "U.S. Listing Gap" or "The Wall Street’s Dead End" has attracted a lot of attention. I build a general equilibrium model to explain there exits a separating equilibrium in which high-growth IPO-qualified firms issue equity in private market while low-growth IPO-qualified firms issue equity in public market. In addition, my models explains the "U.S. Listing Gap" by three major changes during the last two decades: (1) increasing regulatory burden for listing firms without information disclosure function; (2) increasing screening efficiency of private investors; and (3) Increase liquidity in private market. Furthermore, I also partially endogenize the investor’s choice of market for equity investment and show that low-tech investors would not win bidding in private market while high-tech investors can win bidding in some part of the public market, where regulation does not over-protect investors.

1 INTRODUCTION

This paper explains the reduced rate of new listing firms in the US stock market in the last two decades. In the past twenty years, the number of U.S.public firms has declined from 8,025 in 1996 to almost half in 2017, 54% of which is accounted by the reduction in the number of new listing firms (Doidge, Karolyi, & Stulz, 2017). Among the firms that get equity from private markets, there are many very successful unicorns that in the past would go IPO rather then staying private. Famous examples include Uber, Airbnb, Dropbox, Pinterest and so on. When high growth firms choose to be financed through private equity market instead of public equity market, individual investors, who has little access to private market, may suffer\(^1\). The reduced rate of new listing firms thus can increase the wealth inequality and decrease the social welfare. While this question may have important implications about social welfare, researchers are not

\(^1\)https://www.wsj.com/articles/sec-chairman-wants-to-let-more-main-street-investors-in-on-private-deals-1535648208
very clear why many firms choose to get financed through private equity rather than public equity.

To explain such phenomena, I build a general equilibrium model that considers how firm founders choose between public market and private market when they need equity funds. The firm founder has perfect information about the return of their firm while investors in both market only have highly imprecise prior knowledge. The firm founder cares about the ultimate value of their ownership. The public market is subject to regulation so that it has enforced level of information disclosure while the private market is free from regulation and has liquidity discount. Since we focus on the relative ability difference between investors in two market, we assume public investors cannot learn while private investors can optimize their learning with some cost. By Bayesian learning, private investors can get more accurate information about the firm thus require more accurate share they deserve. Investors are same homogeneous within market and across markets in all other dimensions including risk-aversion level, utility function, and reserved utility. Both markets are fully competitive and investors maximize their utility by choosing the share request and learning intensity if they can.

Within this setting, we show that there exists a separating equilibrium in which high-growth IPO-qualified firms issue equity in private market, low-growth IPO-qualified firms issue equity in public market, and some extremely low growth firms are not qualified for IPO.

In addition, my model shows how three major changes in the market induce the decreasing threshold of growth rate that separates the two markets: (1) increasing regulatory burden for listing firms without information disclosure function; (2) increasing screening efficiency of private investors; and (3) Increase liquidity in private market. This decreasing threshold of growth rate actually explains why more promising startups would get finance from private market even though historically similar firms would have gone IPO earlier.

Furthermore, I also partially endogenize the investor’s choice of market for equity investment. I show that, in equilibrium, low-tech investors would not win bidding in private market while high-tech investors can win bidding in some part of the public market, where regulation does not over-protect investors. However, it remains an open question to what extent high-tech investors could win bidding in some part of the public market given they can choose the private market.

2 Literature Review

The last two decades has seen a sharp decline in the number of U.S. public firms (Doidge et al., 2017) and (Doidge, Karolyi, & Stulz, 2013). In 1996, the U.S. has 8,025 domes-
tically incorporated companies listed on a U.S. stock exchange. By the end of 2017, the number is almost only half. This phenomena has draw considerable attention in academia, regulatory agent, and social media. Doidge et al. (2017) call this phenomena ”the U.S. listing gap” and The New York Times describe it as ”The Wall Street’s Dead End”. Below is Figure 1 in Kahle and Stulz (2017) of U.S. number of listed firms and aggregate capital:

![Figure 1](image)

**Figure 1: Regulatory Change**

Doidge et al. (2017) show that, from 1996 to 2012, the high delist rate accounts for 46% of the listing gap and the low new list rate for 54%. Among the delist firms, they show that merger, cause and voluntary account for 59.53%, 37.21% and 3.25% respectively. They further conclude that increased merger account for the majority of increased delist in the post-1996 period.

While there is some research and explanations on the increased delist, not so much is known about the decreased new list, possibly due to the lack of available observations on the IPO-qualified firms that do no choose to go public.

Many regulatory changes during the post-1996 periods have been blamed as contributing to the decline of IPO. Figure 2 below is a summary of recent regulatory
changes from Chart F from the 2011 IPO Task Force (details are explained in below).

This is termed "regulatory overreach hypothesis" by Gao, Ritter, and Zhu (2013). On the one hand, regulatory burden has been increased by some major acts. In particular, Section 404 in Sarbanes-Oxley Act (SOX) in 2002 imposed additional compliance costs on publicly traded firms. The U.S. Securities and Exchange Commission’s (SEC) Regulation FD (Fair Disclosure) in 2000 and the 2003 Global Settlement have also been blamed (see Zweig (2010) and Weild (2011)). Such compliance burden has been especially onerous for small business, and as a consequence, decline in IPOs has been most pronounced among small firms. On the other hand, some researchers attribute the drop in small-company IPO to a worsening situation in the "ecosystem" of underwriters that provide analyst coverage for small firms after IPO. They focus on the shrinking in bid-ask spreads post 1994 and its effect on the declining incentives for small firm coverage. This analyst coverage explanation assumes that, if there is more analyst coverage, the small-firm valuation ratios (e.g., price-to-earnings and market-to-book ratios) are higher. Consistent with this explanation, Jegadeesh and Kim (2009) document that both number of sell-side analysts and the number of firms covered dropped in the post-2002 period.

Such regulatory overreach phenomena has also aroused wide attention from industry and regulatory agent. Many articles in Wall Street Journal and New York Times (see Lucchetti (2011), Salmon (2011), Weild (2011), and Zweig (2010)) have pointed out that the regulatory burden has not only depressed firms from IPO but also pushed domesticate firms to go IPO outside of U.S. Additionally, it has been concluded that the cumulative effect of a sequence of regulatory actions have contributed to the sharp decline of IPO by the 2011 IPO Task Force, a group formed by pro-
professionals representing the entire ecosystem of emerging growth companies – venture capitalists, experienced CEOs, public investors, securities lawyers, academicians and investment bankers. According to a 2007 study by law firm Foley & Lardner (see Lucchetti (2011)), small U.S. public companies’ costs to comply with securities law rose about 1.7 million, roughly 2.8 million a year, after Sarbanes-Oxley passed in 2002. In a 2009 survey conducted by venture capital firm DCM, SOX, corporate governance, and Reg FD were listed as among the top three compliance challenges for small companies thinking of going public. 

In response to growing concerns of regulatory burden of public listing, the U.S. Treasury Department in March 2011 convened the Access to Capital Conference to gather opinions from capital markets participants and solicit recommendations for how to restore access to capital for emerging companies. Among many acts that were passed in the past few years, the 2012 Jumpstart Our Business Startups Act has most impact on reducing regulatory burden for IPO small firms. A recent study by Chaplinsky, Hanley, and Moon (2017) shows that this act only has modest effect on releasing regulatory burden of small listed firms.

Concurrent with the increasing regulatory burden for public listed firms is the consistent deregulation that facilitate firms to raise capital from private financing market, which includes private equity funds, hedge funds and venture capital. One notable deregulation event is the passage of National Securities Markets Improvement Act (NSMIA) in October 1996. NSMIA made it easier for private firms to sell equities to “qualified purchasers” (e.g., institutions or accredited investors) in other states by exempting their private sales from state regulation known as blue-sky laws. (Public sales have long been exempted from the regulation burden of blue-sky laws.) In addition, NSMIA increased maximum number of unregistered funds (e.g., venture capital (VC) and private equity (PE) funds) and exempt them from blue-sky laws. This regulatory change increases the total capital that unregistered funds can raise by involving more investors so that they can meet late-state startups that have higher capital need. Ewens and Farre-Mensa (2018) have identified treatment effect of NSMIA on (1) facilitating late-state startups to raise capital privately and (2) increasing VC and PE funds’ ability in raising more capital. They further claim that the decline in IPO alone is not a market failure, but rather a shift of late-state startups towards private equity market.

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2The fore-mentioned survey results were cited by Kate Mitchell, the National Venture Capital Association chair, in the March 2011 presentation at the U.S. Treasury’s Access to Capital conference.

3One other important deregulation is the SEC’s adoption of Rule 144A in 1990 and subsequent amendments to Rule 144. The ultimate result is that ”Rule 144 now effectively permits the unlimited and unfettered resale of restricted securities (such as private shares) after a six-month or one-year period” (de Fontenay 2017, p. 468). In addition, the Jumpstart my Business Startups (JOBS) Act was signed into law in April 2012, in purpose of reducing the regulatory burden of funding for small firms.
In their survey of CFOs, Brau and Fawcett (2006) conclude managers of successful firms choose to stay private mainly because of their desire to maintain decision-making control and ownership. Chemmanur and Fulghieri (1999), Boot, Gopalan, and Thakor (2006), and Helwege and Packer (2009) also emphasize that founders’ desire to preserve control as a key benefit of remaining private. Our model takes this exactly as the incentive of firm founders.

My general equilibrium models show that there exists a separating equilibrium in which high-growth IPO-qualified firms would like to issue equity in private market while low-growth IPO-qualified firms would like to issue equity in public market. In addition, my models explains the "U.S. Listing Gap" by three major changes during the last two decades: (1) increasing regulatory burden for listing firms without information disclosure function; (2) increasing screening efficiency of private investors; and (3) Increase liquidity in private market. Furthermore, I also partially endogenize the investor’s choice of market for equity investment and show that low-tech investors would not win bidding in private market while high-tech investors can win bidding in some part of the public market, where regulation does not over-protect investors.

My model have the following implications. First, for regulators, they may consider some regulatory easing measure to reduce the IPO burden or compliance burden in order to attract more high-growth firms to choose public financing market. Second, my model partially explains the public firm growth phenomena called "missing skewness" by Decker, Haltiwanger, Jarmin, and Miranda (2016). They show (in Figure 13) that the skewness in growth rates for publicly listed firms increased during the 1990s but has fallen since the early 2000s. Third, regulators may consider to allow public investment access to some good private firms so that (1) public investors could have access to good investment opportunity; (2) investors of private firms can also gain some liquidity of their shares. The Wall Street Journal reports on August 30th 2018 that Jay Clayton, chairman of the Securities and Exchange Commission, said SEC wants to make it easier for individuals to invest in private companies, including some of the world’s hottest startups.4

3 THE MODEL

3.1 Basic Setting

This is a two-period model that an IPO-qualified firm needs equity financing and it chooses between public financing market and private financing market. In period 0, the IPO-qualified firm has exogenous equity financing demand to initiate a new project, with project equity size normalized to 1. To get equity financing, the firm founder has

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to offer some share to the investor in period 0. There are two markets available: the public financing market, which is regulated, and the private financing market, which has no regulation.

Since my model focuses on how the asymmetric information of firm performance and regulatory cost together impact the firm founder’s choice of equity financing, I only need to assume in the model that the firm founder has more information than investor in both markets. Therefore, I assume that, in period 0, the firm founder knows the ultimate return of the project that will be realized and revealed in period 1. However, due to asymmetric information, in period 0, investors from both markets do not know the exact return of the project.

3.2 The Firm and Its Founder

In period 0, the firm founder chooses between the public financing market and private financing market in order to maximize their ultimate share value in period 1. I assume the firm founder is risk-neutral. If he/she chooses the public financing market, he/she will maximize payoff in period 1:

$$V_I = \max [(1 - q_I) \cdot (R_I - C_r)]$$

where \(q_I\) is the share given to the public investor, \(R_I\) is the true return that will be realized in period 1, and \(C_r\) is the regulatory cost. Note that this is a 1 period model so that I do not distinguish flotation cost of new share in the first year of IPO and regulatory compliance cost that would occur every year post IPO. Effectively, the firm founder would choose the public investor who bids with lowest \(q_I\), in order to maximize his/her payoff.

Instead, if the firm founder chooses private financing market, he/she will maximize payoff in period 1:

$$V_P = \max [(1 - q_P) \cdot (R_t)]$$

where \(q_P\) is the share given to the private investor. Effectively, the firm founder would choose the private investor who bids with lowest \(q_P\), in order to maximize his/her payoff.

The firm founder maximize his/her payoff by choosing the higher payoff from the two market, consistent with maximizing control right of the new project. Since the firm founder knows the true return of firm \(R_t\) and the regulatory cost is a common knowledge, effectively the firm founder is maximizing his/her equity share within each market. It is worthwhile to note that my assumptions of a risk-neutral firm founder and maximizing payoff are consistent with the survey of CFOs by Brau and Fawcett (2006), in which managers of successful firms choose to stay private primarily because
of their desire to maintain decision-making control and ownership.

The annual return of the IPO-qualified firms in the market constitute a population of normal distribution \( \mathcal{N}(\mu_0, \sigma_0^2) \). Without any information disclosure, the investors in both market can only guess that the firm is a random draw from the firm population. In other words, the prior knowledge of the firm return is \( R \sim \mathcal{N}(\mu_0, \sigma_0^2) \). I denote the information precision \( h_0 = 1/\sigma_0^2 \).

### 3.3 The Public Financing Market

The public financing market is regulated by government agency. For each unite of IPO, there is a fixed regulatory cost, \( C_r \). This enforced regulation ensures a certain level of information disclosure that any public firm has to obey. Since this is a one-period model, I do not distinguish flotation cost of new shares in the first year of IPO and compliance cost in every year afterwards.

The information disclosure procedure enforced by government agency can be modeled by Bayesian learning with fixed number of information observed. Given the prior knowledge of the firm return \( R \sim \mathcal{N}(\mu_0, \sigma_0^2) \), the public investors learn additional information of the firm by observing a fixed size of noisy information set \( \{X_1, X_2, ..., X_n\} \). Each noisy information is \( X_i = R_t + \epsilon_i \), where \( \epsilon_i \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_0^2) \). I denote the information precision \( h_0 = 1/\sigma_0^2 \). After incorporating the new noisy information with the prior knowledge, investors have updated distribution of firm return:

\[
R_I|\{X_1, ..., X_n\} \sim \mathcal{N}(\mu_I, \sigma_I^2)
\]

where

\[
\mu_I = h_0 * \mu_0 + h_0 * \frac{\sum_{i=1}^{n} x_i}{(n+1)*h_0} = \frac{1}{n+1} * \mu_0 + \frac{n}{n+1} * \bar{x}_n
\]

so \( \mu_I \sim \mathcal{N}(\frac{1}{n+1} * \mu_0 + \frac{n}{n+1} * R_t, \frac{n}{(1+n)^2} * \sigma_0^2) \)

\[
\sigma_I^2 = 1/h_I = \frac{1}{(n+1)*h_0} = \frac{\sigma_0^2}{(n+1)}
\]

Based on updated belief of the firm return, the public investor maximize expected utility of profit:

\[
\max_{0 \leq q_I \leq 1} \mathbb{E}\{U[q_I(R_I - C_r) - 1]\}
\]

subject to \( \mathbb{E}\{U[q_I(R_I - C_r) - 1]\} \geq \bar{u} \)

where \( \bar{u} \) is the reserved utility of public investors that they would like to retain.
3.4 The Private Financing Market

The private financing market has much less regulation than the public financing market but has liquidity discount factor $LD$ in $(0, 1)$. Since I only need to model the relative difference of regulatory burden between above two markets, I assume the private financing market has no regulatory burden. In addition, Private investor can do their research to learn about the distribution of firm return. More importantly, they can choose how much they invest in information learning process, and therefore, determine the level of accuracy of their updated knowledge of the firm return. This information learning process can also be modeled by Bayesian updating in which the private investor can determine the final accuracy of their updated information of the firm return. Given the prior knowledge of the firm return $R \sim \mathcal{N}(\mu_0, \sigma_0^2)$, they can choose how much noisy information they want to learn with cost. Specifically, they determine the size of noisy information \{X_1, X_2, ..., X_m\}. Each noisy observation is $X_i = R_t + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, \sigma_0^2)$. The cost of Bayesian learning $C_P$ is assumed to be a linear function of increase in information precision:

$$C_P = A_P \times \left( \frac{1}{\sigma_P^2} - \frac{1}{\sigma_0^2} \right)$$

where $A_P$ is the cost of a unit increase of information precision.

After determining the size $m$ and incorporating the new noisy information with their prior knowledge, investors have updated distribution of firm return:

$$R_P | \{X_1, ..., X_m\} \sim \mathcal{N}(\mu_P, \sigma_P^2)$$

where

$$\mu_P = \frac{h_0 \cdot \mu_0 + h_0 \cdot \sum_{i=1}^{m} x_i}{(m+1) \cdot h_0} = \frac{1}{m+1} \cdot \mu_0 + \frac{m}{m+1} \cdot x_m$$

so $\mu_P \sim \mathcal{N}(\frac{1}{m+1} \cdot \mu_0 + \frac{m}{m+1} \cdot R_t, \frac{m}{(1+m)^2} \cdot \sigma_0^2)$

$$\sigma_P^2 = \frac{1}{(m+1) \cdot h_0} = \frac{\sigma_0^2}{(m+1)}$$

Based on updated belief of the firm return, the public investor maximize expected utility of profit:

$$\max_{0 \leq q_P \leq 1} \mathbb{E}\{U[LD \cdot q_P R_P - C_P - 1]\}$$

subject to $\mathbb{E}\{U[LD \cdot q_P R_P - C_P - 1]\} \geq \bar{u}$

where $\bar{u}$ is the same reserved utility of public investors that private investors would like to retain.
4 Solve the Model

Based on the model setup, this is a sequential game at period 1 with asymmetric information. I need to first solve the public investor problem and the private investor problem. And then I can solve the firm founder problem. I am going to solve this model by adding some mild assumptions.

4.1 The Public Investor Problem

In order to solve the investors problem in both markets, I need assumption of their utility function. To simplify my model, I assume that investors are homogeneous within each market: they are risk averse, and that their expected utility functions take the form:

\[
\begin{align*}
\mathbb{E}[U(Payoff)] &= \mathbb{E}(Payoff) - \gamma \cdot q_i \cdot \mathbb{V}(R_i), \quad \text{when } \mathbb{E}(\mu_i) > \mu_0 \\
\mathbb{E}[U(Payoff)] &= \mathbb{E}(Payoff) - \text{Leverage} \cdot \gamma \cdot q_i \cdot \mathbb{V}(R_i), \quad \text{when } \mathbb{E}(\mu_i) < \mu_0
\end{align*}
\]

where \(i \in \{I, P\}\) and \(\gamma\) is the risk averse parameter. Higher \(\gamma\) means more risk aversion.

We assume leverage effect in risk aversion because there is empirical evidence of such asymmetric risk aversion and investors can know whether the expected return is higher or lower than average market return. The Leverage is specified as

\[\text{Leverage} = 1 + \text{LevPara} \cdot (\mu_0 - \mathbb{E}(\bar{x}_j))\]

where \(j \in \{n, m\}\) and \(\text{LevPara}\) is a leverage parameter that measures how the risk aversion increases when sample return by Bayesian learning goes far lower than average return of firms. Then the public investor problem is then formulated as

\[
\begin{align*}
\text{when } \mathbb{E}(\mu_I) > \mu_0 & \quad \max_{0 \leq q_I \leq 1} \quad q_I (\mathbb{E}(\mu_I) - C_r) - 1 - \gamma \cdot q_I \cdot \sigma_I^2 \\
& \quad \text{subject to} \quad q_I (\mathbb{E}(\mu_I) - C_r) - 1 - \gamma \cdot q_I \cdot \sigma_I^2 \geq \bar{u}
\end{align*}
\]

\[
\begin{align*}
\text{when } \mathbb{E}(\mu_I) < \mu_0 & \quad \max_{0 \leq q_I \leq 1} \quad q_I (\mathbb{E}(\mu_I) - C_r) - 1 - \text{Leverage} \cdot \gamma \cdot q_I \cdot \sigma_I^2 \\
& \quad \text{subject to} \quad q_I (\mathbb{E}(\mu_I) - C_r) - 1 - \text{Leverage} \cdot \gamma \cdot q_I \cdot \sigma_I^2 \geq \bar{u}
\end{align*}
\]

In addition, I assume that both the public financing market and private financing market are fully competitive. The full competition among public investors effectively forces every public investor choose the same \(q_I\) such that their utility hits their reserved utility \(\bar{u}\):

\[
\begin{align*}
\text{when } \mathbb{E}(\mu_I) > \mu_0 \quad & q_I (\mathbb{E}(\mu_I) - C_r) - 1 - \gamma \cdot q_I \cdot \sigma_I^2 = \bar{u}, \\
\text{when } \mathbb{E}(\mu_I) < \mu_0 \quad & q_I (\mathbb{E}(\mu_I) - C_r) - 1 - \text{Leverage} \cdot \gamma \cdot q_I \cdot \sigma_I^2 = \bar{u}
\end{align*}
\]
Then I can get the solution:

$$
\begin{align*}
q_I &= \frac{\mu + 1}{E(\mu_I) - C_0 - \gamma \sigma_I^2}, \\
q_I &= \frac{\mu + 1}{E(\mu_I) - \text{Leverage} \gamma \sigma_I^2},
\end{align*}
$$

where

$$
E(\mu_I) = \frac{1}{n+1} \mu_0 + \frac{n}{n+1} \gamma R_t
$$

$$
\sigma^2_I = \frac{\sigma_0^2}{n+1}
$$

4.2 The Private Investor Problem

Assuming the same expected utility form as the public investor, the private investor problem is formulated as

When $E(\mu_P) > \mu_0$ then

$$
\begin{align*}
\max_{0 \leq q_P \leq 1, m} & \quad LD * q_P E(\mu_P) - A_P(\frac{1}{\sigma_P^2} - \frac{1}{\sigma_0^2}) - 1 - \gamma * q_P \sigma_P^2 \\
\text{subject to} & \quad LD * q_P E(\mu_P) - A_P(\frac{1}{\sigma_P^2} - \frac{1}{\sigma_0^2}) - 1 - \gamma * q_P \sigma_P^2 \geq \bar{u}
\end{align*}
$$

When $E(\mu_P) < \mu_0$ then

$$
\begin{align*}
\max_{0 \leq q_P \leq 1, m} & \quad LD * q_P E(\mu_P) - A_P(\frac{1}{\sigma_P^2} - \frac{1}{\sigma_0^2}) - 1 - \text{Leverage} \gamma * q_P \sigma_P^2 \\
\text{subject to} & \quad LD * q_P E(\mu_P) - A_P(\frac{1}{\sigma_P^2} - \frac{1}{\sigma_0^2}) - 1 - \text{Leverage} \gamma * q_P \sigma_P^2 \geq \bar{u}
\end{align*}
$$

where

$$
E(\mu_P) = \frac{1}{m+1} \mu_0 + \frac{m}{m+1} \gamma R_t
$$

$$
\sigma^2_P = \frac{\sigma_0^2}{m+1}
$$

Plug above terms into the objective function, then the first order condition gives the optimal $m$:

$$
\begin{align*}
m^* &= \sqrt{\frac{\mu_0 \sigma_P^2}{A_P} \cdot [LD * (E(x_m) - \mu_0) + \gamma \sigma_0^2]} - 1, \quad \text{when } E(\mu_P) > \mu_0 \\
m^* &= \sqrt{\frac{\mu_0 \sigma_P^2}{A_P} \cdot [LD * (E(x_m) - \mu_0) + \gamma \sigma_0^2]} - 1, \quad \text{when } E(\mu_P) < \mu_0
\end{align*}
$$

The second order condition

$$
\begin{align*}
-2 * \frac{A_P}{(m+1) \sigma_0^2} & \leq 0, \quad \text{when } E(\mu_P) > \mu_0 \\
-2 * \frac{A_P}{(m+1) \sigma_0^2} & \leq 0, \quad \text{when } E(\mu_P) < \mu_0
\end{align*}
$$

confirms that above $m^*$ is indeed the maximizer of the expected payoff.

Then the fully competition assumption forces every private investor to choose $q_P$ such that their expected utility hits the reserved utility. In this way, I get the numeric solution of $q_P$.
4.3 The Firm Founder Problem

The assumption of homogeneous investor within each market essentially enforces investors in the same market request the same share so that firm founder cannot maximize their own value within each market. Instead, they can still maximize value between two markets. Anticipating the equilibrium results of $q_I$ and $q_P$, the firm founder chooses between

$$
\begin{align*}
V_I &= (1 - q_I) * (R_t - C_r) \\
V_P &= (1 - q_P) * (R_t)
\end{align*}
$$

5 Separating Equilibrium

Now I use simulation to generate my main results: Separating Equilibrium. I set $\bar{u} = 0$ to be consistent with the fact that investors can use treasury note as their alternative investment, where volatility is almost zero and I normalize interest rate to be zero. I set $C_r = 0.09$ according to Curragh, Leveque, and Dhar (2012), gross return of average IPO $\mu_0$ as 1.4, unconditional volatility 0.4, risk aversion parameter $\gamma$ as 1, IPO market mandatory information disclosure intensity $n$ as 1, and Bayesian learning unit cost $A_P = 0.002$. I can ge the following separating equilibrium that high return (growth) IPO-qualified firms would prefer private financing market while low return (growth) IPO-qualified firms can only go IPO. This is demonstrated in figure 3.

![Separating Equilibrium: Value of Firm Founder](image)

Figure 3: Separating Equilibrium: Value of Firm Founder
The corresponding regulatory information disclosure intensity and information learning intensity by private investors are demonstrated in figure 4.

Figure 4: Separating Equilibrium: Information Disclosure and Learning Intensity
6 Comparative Statics

Next, I show that three major channels can reduce the critical $R_t$ that separates IPO-market and private market:

- Increased regulatory burn without information disclosure function;
- Decreased information learning unit cost by private investor;
- Increased liquidity of private financing market.

6.1 Increased regulatory burn without information disclosure function

In my model, inefficient increase in regulatory burden can be modeled by increased regulatory burn without information disclosure function. In particular, $C_r$ increases without increased information disclosure intensity $n$. This change will result in increased inefficiency of IPO market relative to private market, and therefore, decreased critical $R_t$ that separates IPO-market and private market. This is demonstrated in Figure 5.

![Figure 5: Separating Equilibrium: When $C_r$ changes](image)

6.2 Decreased information learning unit cost

In my model, decreased information learning unit cost can be modeled directly by decreased $A_P$. This change will result in increased information learning intensity $m_{opt}$
by private investor, and thus decreased critical $R_t$ that separates IPO-market and private market. This can be seen from figure 6.

![Figure 6: Separating Equilibrium: When $A_p$ changes](image)

Figure 6: Separating Equilibrium: When $A_p$ changes

### 6.3 Increased Liquidity of Private Financing Market.

In my model, increased liquidity in private financing market can be modeled directly by increased liquidity discount factor $LD$ so that private market suffers from less liquidity discount. This change will result in overall increased $V_P0$ and this can be seen from figure 7.
7 Endogenize Investor’ Choice of Market for Investment

Up to this section, I only assume that high-tech investors go to private financing market and low-tech investors go to IPO market. In this section, I am going to show that low-tech investor can only go to IPO market. However, high-tech investors can go to IPO market and succeed only in bidding for high-growth firms.

7.1 Low-Tech Investor Would Not Go To Private Market

First, assume low-tech investors go to private financing market. Since they cannot learn and therefore cannot distinguish firms that are better than IPO-firms and firms that perform lower than IPO-forms, they can only use their prior knowledge. Therefore, without knowing more information, they only charge very high share to protect themselves, thus unable to compete with private investors. This can be illustrate in figure 8.

7.2 High-Tech Investor Can Go To Some Part of Part of IPO Market

More interesting findings come out when high-tech investors go to IPO market. In IPO market, firms undertake the IPO cost and then disclose what information is re-
required. Since high-tech investors can optimize their learning process, they can win the bids with lower share request when IPO-mandatory information disclosure is lower than their optimal one. This is the situation in Figure 9 when \( R_t \geq 1.28 \). However, when the IPO-mandatory information disclosure intensity is higher than their optimized one, they cannot win the bid since they incur additional cost for their baseline learning. Baseline learning is defined as the minimum learning intensity that high-tech investors must choose in order to decide whether to learn or not beyond the prior knowledge. This can be illustrate in figure 9 when \( R_t < 1.57 \) in the previous IPO market.

Above findings can be easily understood when I look at the figure 10 when I compare the information disclosure and learning intensity with the difference between firm founder value when private investors go to IPO market \( (V_{PI}) \) and the one when IPO investor bid in the IPO market \( (V_I) \).

In this figure, the green dashed line is the optimal learning intensity of privator investors when they bid in private market. However, when they go into IPO market, due to the mandatory information disclosure intensity \( n \), they can only learn by their additional cost for their baseline learning in the range of \( R_t : (1.29, 1.57) \). After such baseline learning, they know that it is better for them to stop learning since the marginal cost is less the marginal benefit. Therefore, in this range, they occur additional learning cost than the low-tech investors and request higher share. As a result, they cannot compete with low-tech investors within this range. However, in the \( R_t \) range of \( (1.27, 1.29) \) and \( (1.57, + \text{inf}) \), high-tech investors can optimize their learning
to achieve higher learning intensity then low-tech investors. In this way, they can know more accurate information with low cost thus requiring lower share. As a consequence, they win the bidding within this two ranges.
Figure 10: Separating Equilibrium: When $LD$ changes
References


