Advertising Auctions*

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Abstract

We introduce informative advertising into an optimal auction problem with endogenous entry. Specifically, we consider an independent-private-values environment with costly participation and allow the seller to choose not only the reserve price, but also how much product information to provide for bidders before they enter. The revenue-maximizing information structure takes a binary-cutoff form; the seller informs each bidder whether his value is above or below a threshold. We study how the level of this threshold and the optimal reserve price depend on the entry cost. We show that the seller can extract full surplus if and only if the entry cost is sufficiently large and that the optimal reserve price necessarily decreases in the entry cost if and only if the number of potential bidders is sufficiently small.

JEL Classification Numbers: D11, D21, D44, D61, D82, D83, M37.

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1 Introduction

It is well-known that costly participation (or endogenous entry) of bidders changes several canonical predictions for auction outcomes. In particular, with endogenous entry, the seller wants to set a lower reserve price than in the usual case with exogenous entry (i.e., a fixed set of bidders), and it is often optimal for the seller to run an efficient auction (i.e., set the reserve price equal to his own valuation). Intuitively, this is because the seller must provide an incentive for potential bidders to incur the cost to enter the auction. In addition, if she is a residual claimant after bidders' entry costs and information rents, it is optimal for her to maximize social surplus. This insight has been

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proven to be robust in various environments (e.g., Levin and Smith, 1994; Peters and Severinov, 1997; Kim and Kircher, 2015).

We introduce another instrument, *informative advertising*, into this optimal auction problem. Specifically, we consider the standard independent-private-values environment with endogenous entry and allow the seller to choose not only the reserve price but also how much, and which, information to reveal about the object for sale before bidder entry. In our model, (ex ante homogeneous) bidders learn their (true) values for the object once they incur the fixed cost c and enter the auction.[1] Initially, they are uninformed about their values, but the seller can provide information for them. This means that the seller can influence bidders' participation both by adjusting the reserve price, as widely studied in the literature, and by controlling bidders' pre-entry information.

Following the burgeoning literature on information design, we endow the seller with full flexibility in advertising content. In other words, we impose no structural restriction on the set of the seller's feasible advertising strategies and allow her to reveal (or conceal) whatever information she wants.[2] The seller may choose to provide no information (in which case each bidder decides whether to enter or not based on her prior), full information (in which case each bidder perfectly observes his value before making an entry decision) or anything in between: for example, she may interval-partition the set of feasible values and only inform each bidder which interval his value belongs to. As is common in the information-design literature, this full flexibility enables us to separate the seller's fundamental incentives from any ad-hoc assumptions about information structures.

At a technical level, our investigation can be understood as an attempt to endogenize bidders' (pre-entry) information in auctions with entry.[3] Recall that most previous studies on IPV auctions with entry assume either that bidders are completely uninformed about their values before entry (e.g., McAfee and McMillan, 1987; Levin and Smith, 1994) or that they are fully informed (e.g., Samuelson, 1985; Engelbrecht-Wiggans, 1993; Matthews et al., 1995).[4] As explained above, these two extreme cases are always available to the seller but, as shown later, they never arise as the seller's optimal choice. This implies that our analysis informs the extent to which existing insights from the literature on auctions with entry depend on such strong informational assumptions.

Although it has not been well-studied in the auction literature, advertising (providing infor-

[1] In this sense, we study the case of *search goods* (for which bidders necessarily discover their true values upon inspection), not that of *experience goods* (for which bidders find out their true values only after bidding). For the latter type of information-design problems, see, among others, Bergemann and Pesendorfer (2007), Bergemann, Brooks and Morris (2017), and Roesler and Szentes (2017).

[2] In this paper, we restrict attention to *anonymous* and *independent* information structures. See Section 2 for a detailed discussion.

[3] In this regards, our paper is related to work on endogenous information acquisition by bidders. See, e.g., Bergemann and Välimäki (2002), Persico (2000), Compte and Jehiel (2007), Rezende (2018), and Shi (2012).

[4] There are a small number of papers which assume that bidders possess partial information before participation. See, e.g., Ye (2007), Lu and Ye (2014), and Quint and Hendricks (2018)).

mation about the auctioned object) is prevalent and plays an important role in real auctions. For example, one of the most important roles of auction houses, such as Christie's and Sotheby's, is to generate the right amount of interest (competition) for each auction item by maintaining the pool of potential bidders and supplying them with relevant information (e.g., through catalogues). Another example is the fast-growing market for online advertisements: websites and applications on smart devices auction off space on their platforms to advertisers through real-time bidding. When a consumer visits a webpage or opens an application, she triggers an ad request. Some potential advertisers are contacted and told about the request. If an advertiser decides to participate in the auction (which because of computing power and marketing campaign budgets involves some opportunity cost), then more information about the consumer is passed along to him.

We first show that the revenue-maximizing information structure takes a simple binary-cutoff structure, that is, the seller cannot do better than informing each bidder whether or not his value is above a certain threshold $\tilde{v}$. In order to understand this result, notice that, while lowering the reserve price always encourages bidder participation, the effect is not monotone with information: more information makes a bidder more likely to participate when his true value is relatively high but less likely to enter when his true value is relatively low. This trade-off induces an ambiguous relationship between no information and full information: the former yields higher revenue than the latter when c is small, while the opposite holds when c is sufficiently large. In our model with endogenous information, however, neither of them is optimal, that is, the trade-off is optimally resolved by an interior information structure that provides some, but not all, information for bidders. Specifically, the optimality of a binary information structure stems from the fact that information (advertising) affects only bidders' participation decisions, which are binary. The optimality of the cutoff structure is driven by the fact that the higher a bidder's value is, the more his participation contributes to social surplus in expectation and, therefore, the seller's revenue as well.

Given the optimal information structure, we characterize the corresponding optimal reserve price r^* and how the revenue-maximizing mix of $\tilde{v}$ and r^* depend on entry costs c. We show that if c is sufficiently large, then the seller can extract full surplus (which coincides with the amount she can obtain with entry fees): our analysis demonstrates that this full-surplus extraction is possible exactly because the seller can simultaneously adjust both the reserve price r and the information cutoff $\tilde{v}$, not just one of them. If c is sufficiently small, then it is optimal for the seller to set the same reserve price as in the case with exogenous entry and induce all bidders to participate: the seller can achieve the same revenue with no information, but not with full information. Finally, if c is neither too large not too small, then the seller's optimal choice of r depends on the number of potential bidders. If the number of bidders is small, then as c increases, the seller lowers both the information cutoff $\tilde{v}$ and reserve price r to maintain attendance. If the number of bidders is large, then the seller raises the information cutoff and reserve price as c increases.

Besides the literature on auctions with endogenous entry, our paper is most closely related to Anderson and Renault (2006). They analyze the optimal monopoly problem when the monopolist can set a (uniform) price and choose how much product information to reveal before consumers make a search (visit) decision. Just as in our paper, they impose no structural restrictions on feasible advertising content, investigate the seller's optimal mix of advertising and pricing, and find that the optimal advertising strategy takes a simple binary-cutoff structure.[5] The main difference from our paper is that the seller faces no capacity constraint, and thus there is no competition among buyers: our model shrinks to theirs if there is only one potential bidder. As explained in Section 4.1, our main technical challenge is to incorporate competition among buyers into participation decisions: note that in our model, a bidder's participation incentive depends not only (directly) on the seller's choice of reserve price and advertising, but also (indirectly) on other bidders' reactions to the seller's strategy. Our results can be interpreted as demonstrating the robustness of their main insights, while qualifying some of their specific results,[6] in terms of buyer competition.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 considers the benchmarks of no information, full information, and social efficiency. In Section 4, we analyze the seller's optimal choices of information structure and advertising, and in Section 5, we conclude with a discussion of revenue comparison and avenues for future research.

2 Model

A seller wishes to sell an indivisible object, whose value to her is normalized to 0. There are $N(\geq 2)$ risk-neutral *potential* bidders, whose values for the object are independently and identically drawn according to a distribution function F with support $\mathcal{V} \equiv [0, \overline{v}] \in \mathcal{R}_+$. We assume that F has a continuously differentiable density f and that its hazard rate $f/(1-F)$ is strictly increasing on $\mathcal{V}$. The latter assumption ensures that there is a unique interior solution r_0 that maximizes $(1-F(r))r$ and that r_0 is fully characterized by the relevant first-order condition $r_0 = (1-F(r_0))/f(r_0)$. Note that r_0 is the optimal reserve price in the standard IPV auction (see, e.g., Krishna, 2009).

As in Levin and Smith (1994), bidders learn their true value v once they decide to participate in the auction. Specifically, each participating bidder incurs a fixed and irrevocable cost $c > 0$. In order to avoid triviality, we maintain the assumption that $c < \overline{v}$. This cost c can be interpreted as a transportation cost of attending the auction, an information processing cost of assembling a

[5] Choi, Kim and Pease (2019) study the buyer-optimal and the seller-worst signals in the same environment as Anderson and Renault (2006). They show that those signals take a starkly different structure from the seller-optimal signal in Anderson and Renault (2006).

[6] For example, in Anderson and Renault (2006), the optimal price always decreases in search costs c. As explained above, this is not necessarily the case in the presence of buyer competition: if the number of potential bidders is sufficiently large, then the optimal reserve price may increase in c over an intermediate range.

bid, or the opportunity cost of not attending a different auction if the bidder is time- or budget-constrained. Once a bidder pays c, he perfectly learns his value v and can submit a bid. We focus on the case where the seller runs a second-price auction (in which each bidder plays a weakly dominant strategy of bidding his true value v) and chooses a reserve price r.[7] General revenue equivalence (e.g., Che and Gale, 2006) implies that our analysis applies unchanged even if the seller adopts a different standard auction.[8]

Unlike Levin and Smith (1994), the seller can provide information about the object for sale before bidders make their entry decisions. In other words, the seller can influence bidders' participation through informative advertising. Specifically, we assume that the seller chooses an arbitrary set of signal realizations, S, and a joint distribution π over $\mathcal{V} \times S$. For notational simplicity, we use $\pi(s|v)$ to denote the probability that the realized signal is s conditional on true value v. Each bidder observes the signal structure (S, π), receives a signal realization $s \in S$, updates his belief about v, and then decides whether to enter the auction or not.

As in Anderson and Renault (2006), our model gives the seller full flexibility in her choice of advertising content. For example, fully informative advertising corresponds to the case where $S = \mathcal{V}$ and $\pi(v) = v$ for all $v \in \mathcal{V}$. Fully uninformative advertising (equivalently, no advertising) is when S is a singleton. The seller can also provide partial information by sending the same signal to multiple types of bidders. Note, however, that each bidder's signal s can depend only on his value v, not on other bidders' values: this excludes, e.g., the possibility that the seller informs a bidder that he is one of the top 2 bidders. In addition, the same signal structure (S, π) applies to all bidders, which indicates that the seller cannot play a discriminatory advertising strategy (of providing different information to different bidders). We impose these restrictions in order to capture the reality that advertising provides information about the object for sale and cannot reflect recipients' reactions to it. We discuss these restrictions further in Section 5.

The overall timing of the game is as follows. First, the seller chooses an information structure (S, π) and a reserve price r. Second, each bidder observes both (S, π) and r, receives a signal $s \in S$ according to π, and then decides whether to incur entry cost c or not. Third, each participating bidder observes his value v and submits a bid. Finally, the winner is determined according to the second-price auction with reserve price r. The players' payoffs, depending on their participation decisions and the second-price auction outcome, are straightforward and, therefore, omitted.

[7] One advantage of the second-price auction is that bidders' equilibrium bidding strategies are independent of other bidders' behavior and, therefore, it is not required to specify bidders' knowledge about other participating bidders.

[8] Che and Gale's revenue equivalence applies to first- and second-price sealed bid auctions, all pay auctions, and wars of attrition, among others, but not to third-price auctions.

3 Benchmarks

This section presents certain benchmark results for our main analysis in Section 4. We first consider two extreme cases in which the seller provides either no information or full information about her object. This will not only highlight how pre-search information affects bidders' participation but also provide lower bounds for the seller who can freely choose how much and what information to provide. We then analyze the case in which the seller can use entry fees/subsidies and, therefore, extract full surplus. Certainly, this will provide an upper bound for the seller's revenue in our main model.

3.1 No Information

Suppose that bidders receive no information before making participation decisions; either the seller decides not to provide any information or she is not allowed to transmit any information. Then, our model reduces to the IPV case of Levin and Smith (1994), in which bidders typically play a symmetric mixed entry strategy (which constitutes an equilibrium when c is neither too low nor too high). Below, we provide a slightly more general characterization, covering the entire range of c and characterizing the optimal reserve price r as a function of c, for the current IPV environment.

Let $u_N(r, q)$ denote a participating bidder's expected payoff when the reserve price is r and every other bidder enters the auction with probability $q \in [0, 1]$. In a second-price auction, the bidder wins the object if and only if his value v exceeds r and all other bidders either do not enter or have values lower than v. Conditional on winning, he pays either the reserve price r or the second-highest value. Letting $H_q(v) \equiv (1 - q + qF(v))^{N-1}$ denote the probability that all other bidders either do not enter or draw values less than v conditional on q, we have

$$u_N(r, q) = \int_r^{\overline{v}} \left(vH_q(v) - rH_q(r) - \int_r^v x dH_q(x) \right) dF(v) = \int_r^{\overline{v}} H_q(v)(1 - F(v))dv,$$

It is straightforward to show that $u_N(r, q)$ decreases in both r and q; a bidder earns more surplus from attending the auction the lower the reserve price is or the less likely the other bidders are to enter the auction.

Given c and r, the equilibrium value of q (the probability that each bidder enters the auction) is determined as follows:

- If $c \leq u_N(r, 1)$, then $q = 1$, that is, all the bidders enter the auction.
- If $c \geq u_N(r, 0)$, then $q = 0$, that is, no bidder enters the auction.
- If $c \in (u_N(r, 0), u_N(r, 1))$, then $q \in (0, 1)$, that is, all the bidders randomize between entering the auction and staying out.

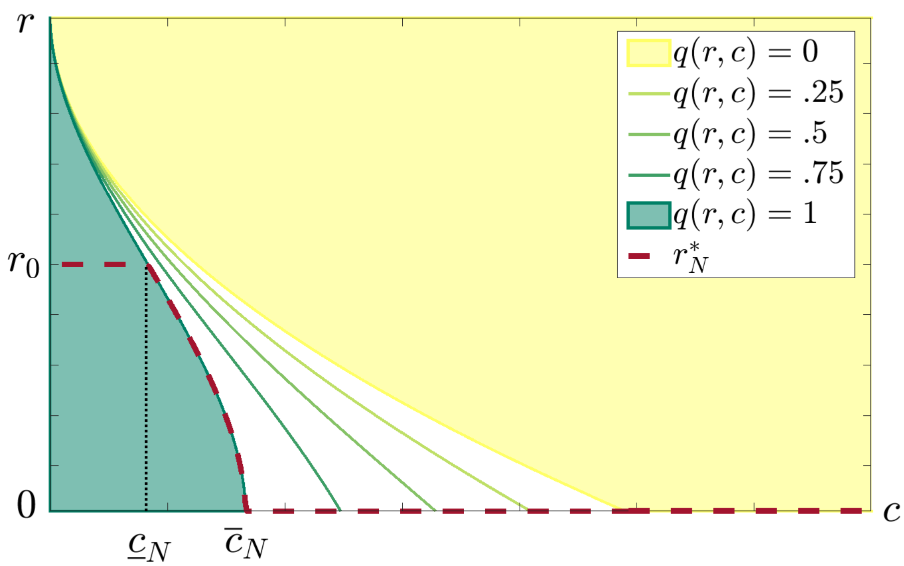

Figure 1: Using the uniform distribution, each solid line depicts a level curve that corresponds to a $q \in [0,1]$, that is, the set $\{(c,r) : q(c,r) = q\}$ for each q. The thick dashed curve is the revenue-maximizing reserve price r_N^* as a function of c.

In the last case, the equilibrium value of q satisfies $c = u_N(r,q)$. Such a value uniquely exists, because $u_N(r,q)$ is continuous and strictly decreasing in $q \in (0,1)$. The following result is also straightforward from the fact that $u_N(r,q)$ is decreasing in r.

Lemma 1 *Let $q(c,r)$ denote the probability that each bidder enters the auction in the model with no informationprior to attendance. Then, $q(c,r)$ decreases in both c and r.*

Figure 1 visualizes Lemma 1. If both c and r are sufficiently small, then bidders expect a large surplus from the auction, and thus all of them participate; this corresponds to the fact that $q(c,r) = 1$ in the southwest corner in Figure 1. As either c or r increases, bidder surplus decreases, and thus they are less likely to enter the auction; in the middle region, the color of the level curves become lighter (i.e., $q(c,r)$ decreases), as c or r increases. If c becomes sufficiently large or the seller sets the reserve price too high, then no bidder enters the auction; $q(c,r) = 0$ in the northeast region.

Given c and r (and the resulting equilibrium participation behavior $q(c,r)$), the seller's expected revenue, which is the sum of all bidders' expected payments, is given by

$$\begin{aligned} V_N(c,r) &\equiv Nq \int_r^{\overline{v}} \left(rH_{q(c,r)}(r) + \int_r^v x dH_{q(c,r)}(x) \right) dF(v) \\ &= Nq \left(rH_{q(c,r)}(r)(1-F(r)) + \int_r^{\overline{v}} v(1-F(v)) dH_{q(c,r)}(v) \right). \end{aligned}$$

The seller's problem is to choose r that maximizes $V_N(c, r)$ given c. The following proposition characterizes the optimal reserve price as a function of c.

Proposition 1 *Let $r_N^*(c)$ denote the seller's optimal reserve price in the model with no information prior to attendance. There exist $\underline{c}_N(> 0)$ and $\overline{c}_N(> \underline{c}_N)$ such that*

(i) if $c \leq \underline{c}_N$ then $r_N^*(c) = r_0$,

(ii) if $c \in (\underline{c}_N, \overline{c}_N)$ then $r_N^*(c)$ is such that $q(c, r_N^*(c)) = 1$, and

(iii) if $c \geq \overline{c}_N$ then $r_N^*(c) = 0$.

Proof. See Appendix. ■

The dashed line in Figure 1 illustrates Proposition 1. In the standard model with no participation cost (i.e., as in Myerson, 1981), the seller optimally sets a reserve price equal to r_0. When c is sufficiently small, this continues to be the best reserve price to set because all bidders are participating ($q = 1$), giving the seller the same expected revenue as in the problem without bidder entry. If c passes a certain level ($\underline{c}_N$), then r_0 discourages bidder participation (that is, $u_N(r_0, 1) < c$). In this case, the seller faces a trade-off: a higher reserve price enables her to extract more from participating bidders but inhibits bidders' participation. Then, the optimal reserve price is the level at which all bidders have a just enough incentive to enter the auction, that is, where $u_N(r, q(c, r)) = c$; this leaves no rents to the bidders, while maximizing their participation. If c becomes sufficiently large, then it is optimal to set $r = 0$. This can be interpreted as the case in which $u_N(r, 1) < c$ for any $r \geq 0$ and, therefore, it is impossible to induce full bidder participation. Then, as is well-known, it is optimal to run an efficient auction, because bidders never get positive surplus and, therefore, the seller's expected revenue coincides with social surplus. Finally, if cost is extremely high, then even a reserve price of 0 results in no participation.

3.2 Full Information

Now suppose that bidders receive full information before deciding whether to enter the auction or not. In this case, obviously, the higher a bidder's value, the stronger his incentive to enter the auction. Therefore, bidders' equilibrium entry strategies take a simple form: each bidder enters the auction if and only if his value exceeds a certain threshold, which we denote by $v(c, r)$.

In order to characterize the equilibrium $v(c, r)$, let $u_F(v, r, v')$ denote a bidder's expected payoff when he enters the auction with value v, the reserve price is r, and every other bidder enters the auction if and only if his value exceeds v'. Clearly, if $v \leq r$ then the bidder has no incentive to participate. Therefore, we consider only the case where $v, v' \geq r$. Letting

$H_{v'}(v) \equiv (F(v') + \max\{F(v) - F(v'), 0\})^{N-1}$ denote the probability that all other bidders either do not participate (less than v') or have values less than v, $u_F(v, r, v')$ is given by

$$u_F(v, r, v') = vH_{v'}(v) - rH_{v'}(v') - \int_{v'}^{\max\{v,v'\}} x dH_{v'}(x).$$

It is straightforward to show that $u_F(v, r, v')$ increases in v and v' and decreases in r; a bidder obtains a higher expected payoff, the higher his value is, the less likely other bidders are to participate, or the lower the reserve price is.

Given the other bidders' participation cutoff v', a bidder enters the auction if and only if his expected payoff $u_F(v, r, v')$ by doing so exceeds entry costs c. In addition, in the symmetric equilibrium, his optimal cutoff must coincide with the other bidders' cutoff v'. These imply that the equilibrium cutoff $v(c, r)$ satisfies

$$u_F(v(c, r), r, v(c, r)) = c.$$

In other words, $v(c, r)$ is such that a bidder with value $v(c, r)$ breaks even by entering the auction conditional on the event that every other bidder participates if and only if his value exceeds $v(c, r)$. From the monotone properties of $u_F(v, r, v')$, the following result is straightforward.

Lemma 2 *Let $v(c, r)$ denote the lowest bidder value that is willing to enter the auction in the model with full information prior to attendance. Then, $v(c, r)$ increases in both c and r.*

Figure 2 shows the behavior of $v(c, r)$. Buyers are more willing to enter the auction, the lower search costs c and the reserve price r are. Therefore, $v(c, r)$, the lowest participating bidder type, decreases as either c or r decreases. This implies that the level set $\{(c, r) : v(c, r) = v\}$ is downward-sloping, as shown in the figure.

Given c and r (and the resulting equilibrium participation cutoff $v(c, r)$), the expected payment of a bidder with $v(\geq v(c, r))$ is equal to

$$p(v; c, r) \equiv rH_{v(c,r)}(v(c, r)) + \int_{v(c,r)}^{v} x dH_{v(c,r)}(x),$$

because the bidder wins only when all other bidders' values are less than v (so, with probability $H_{v(c,r)}(v)$), and he pays r if no bidder participates (the first term) and the second-highest value

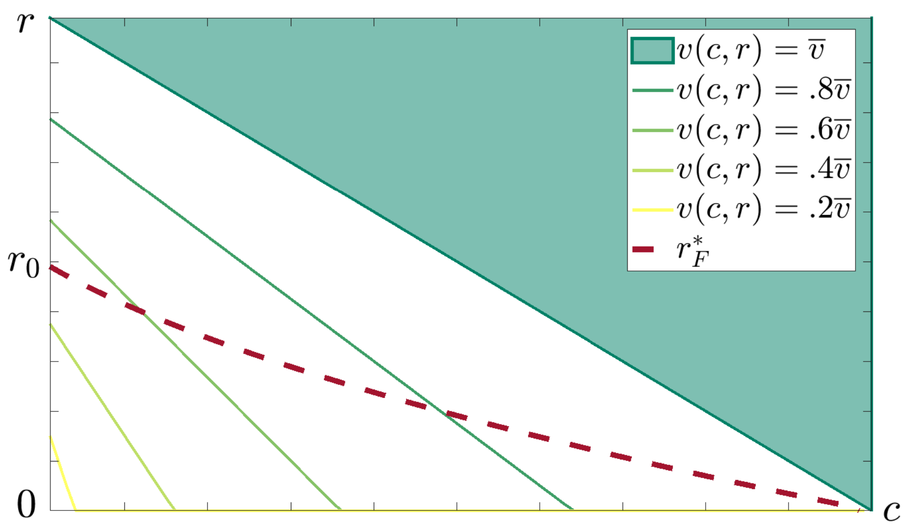

Figure 2: Using the uniform distribution, each solid line represents a level curve that corresponds to a $v \in [0, \overline{v}]$, that is, the set $\{(c, r) : v(c, r) = v\}$ for each v. The thick dashed curve denotes the revenue-maximizing reserve price r_F^* as a function of c.

otherwise (the second term). Therefore, the seller's total expected revenue is given by

$$\begin{aligned} V_F(c,r) &= N \int_{v(c,r)}^{\overline{v}} p(v;c,r)dF(v) \\ &= N\left(r(1-F(v(c,r)))H_{v(c,r)}(v(c,r)) + \int_{v(c,r)}^{\overline{v}} v(1-F(v))dH_{v(c,r)}(v)\right). \end{aligned} \tag{1}$$

The following proposition explains how the reserve price that maximizes $V_F(c, r)$ varies according to c.

Proposition 2 *Let $r_F^*(c)$ denote the seller's optimal reserve price in the model with perfect information prior to participation. Then, $r_F^*(c)$ is continuously decreasing in $c \in (0, \overline{v})$.*

Proof. See the appendix. ■

The dashed line in Figure 2 exemplifies the behavior of $r_F^*(c)$. For c sufficiently close to 0, the seller does not need to provide an incentive for bidders to enter, in which case it is optimal for her to choose the same optimal reserve price as in the case without entry (that is, $r_F^*(0) = r_0$). As c rises, however, bidders become less willing to participate, thereby adjusting their participation cutoff $v(c, r)$ upwards. The seller can counter and mitigate this bidder reaction by lowering her reserve price, and Proposition 2 shows that it is indeed optimal for her to do so. For any $c(< \overline{v})$, it is optimal to set a strictly positive reserve price, but $r_F^*(c)$ approaches 0 as c tends to $\overline{v}$ (because otherwise no bidder would enter the auction).

3.3 Full Surplus Extraction through Entry Fees

The analysis so far has characterized the optimal reserve prices given two extreme information structures. In order to evaluate their performances (and that of our main model in which the seller can choose a signal structure) from the seller's perspective, we now characterize the maximum amount of surplus available in our environment. We also show that if the seller can impose entry fees or provide entry subsidies, then she can always extract full surplus by using a simple binary signal, which informs each bidder of whether his value exceeds a certain threshold or not.

Consider a social planner who wishes to maximize social surplus but faces the same informational constraint as in our model. Specifically, suppose that the social planner can impose any symmetric entry rule on the bidders, that is, she can choose any function $\gamma : [0, \overline{v}] \to \{0, 1\}$, where 0 represents "do not enter", while 1 represents "enter." Given a function γ, each bidder enters if and only if his value v belongs to $\gamma^{-1}(1)$.[9] Since entry costs are independent of a bidder's type, it is clear that an optimal entry strategy is a simple cutoff rule such that each bidder enters the auction if and only if his value exceeds a certain threshold v^*. It is optimal to assign the object to the highest bidder among those who participate. These observations imply that the social planner's problem can be written as follows:

$$\max_{v^* \in [0,\overline{v}]} \int_{v^*}^{\overline{v}} v dF(v)^N - N \int_{v^*}^{\overline{v}} c dF(v).$$

It is straightforward to obtain the following result:

Proposition 3 *Let $v^*(c) \in [0, \overline{v}]$ be the unique value of v such that $vF(v)^{N-1} = c$. Social surplus is maximized when each bidder enters the auction if and only if his type exceeds $v^*(c)$ and the object is allocated to the highest type bidder.*

To gain intuition, note that the lowest participating type v^* contributes to social surplus if and only if no other bidder draws a higher value than v^* and, therefore, enters the auction. This implies that his marginal social contribution is equal to $v^* F(v^*)^{N-1}$. Clearly, it is socially optimal that a bidder participates if and only if his marginal social contribution by doing so exceeds the

[9] Note that the social planner is restricted to apply the same entry rule (that depends only on each bidder's type) to all bidders. This corresponds to the requirement in our model that the seller chooses a common signal for all bidders, that is, she is not allowed to correlated bidders' signals. If the social planner does not face this constraint, she can achieve the first-best outcome such that only the highest bidder type enters conditional on his value exceeding c.

corresponding cost c. Therefore, at the socially optimal participation cutoff $v^*(c)$, it must be that $v^*(c)F(v^*(c))^{N-1} = c$. Note that $v^*(c)$ is strictly increasing in c and approaches $\overline{v}$ as c tends to $\overline{v}$.

Now we demonstrate that the seller can always extract full surplus if she can choose both an entry fee and an information structure. Specifically, suppose that the seller can charge each participating bidder an entry fee e, which can be either positive or negative. In addition, suppose that the seller adopts the following simple binary signal structure: $S = \{0, 1\}$ and $\pi(v) = 1$ if and only if $v \geq v^*(c)$. In other words, the seller provides partial product information, so that each bidder learns only whether his value is above or below $v^*(c)$.

Let $u(s)$ denote a bidder's expected payoff when he enters the auction after observing signal $s \in \{0, 1\}$. If $s = 1$, then he knows that his value is above $v^*(c)$. Since every other bidder enters if and only if his value exceeds $v^*(c)$,[10] his expected payoff by entering the auction (excluding entree fees) is given by

$$\begin{aligned} u(1) &\equiv \frac{1}{1 - F(v^*(c))} \int_{v^*(c)}^{\overline{v}} \left(vH(v) - \int_{v^*(c)}^{v} x dH(x) \right) dF(v) \\ &= c + \frac{1}{1 - F(v^*(c))} \int_{v^*(c)}^{\overline{v}} (1 - F(v))H(v)dv, \end{aligned}$$

where $H(v) \equiv F(v)^{N-1}$. It is trivial to show that $u(0) < u(1)$. The following result is then straightforward.

Proposition 4 *If the seller can impose entry fees/subsidies, she can extract full surplus by using a simple binary signal that informs each bidder of whether his value is above or below $v^*(c)$ and setting*

$$e = u(1) - c = \frac{1}{1 - F(v^*(c))} \int_{v^*(c)}^{\overline{v}} (1 - F(v))H(v)dv.$$

Proof. Given $e = u(1) - c$ (and assuming that each other bidder enters if and only if $s = 1$), each bidder has a just right incentive to enter the auction conditional on $s = 1$ (i.e., $u(1) - (c + e) = 0$) and strictly prefers not entering conditional on $s = 0$ (i.e., $u(0) - (c+e) < 0$). Then, social surplus is maximized because a bidder receives $s = 1$ if and only if his value exceeds $v^*(c)$ (so, efficient participation) and the highest-value bidder, conditional on exceeding $v^*(c)$, always receives the object (so, efficient allocation). $e = u(1) - c$ also ensures that each bidder's expected payoff is equal to 0. It then follows that the seller extracts full surplus. ■

The fact that the seller can extract full surplus with entry fees is not surprising at all, as it is a

[10] It is trivially the case that a bidder has a stronger incentive to participate when $s = 1$. Therefore, for the signal structure to influence bidders' participation decisions, it must be that each bidder enters the auction if and only if $s = 1$.

direct extension of the usual two-part tariff.[11] Proposition 4, however, highlights how much *more* surplus the seller can extract through advertising (i.e., by controlling pre-attendance information). To see this more clearly, reconsider the no-information case in Section 3.1, but now suppose that the seller can use the entry fee. It is easy to see that the seller can, again, extract full surplus *subject to the (no-)information constraint*.[12] In other words, even though she can always take away bidder surplus, she cannot do as well as in Proposition 4, because she is constrained by the fact that bidders receive no information. Providing some information can make bidder entry more efficient, thereby increasing social surplus and allowing the seller to extract even more surplus.

4 Main Characterization

We now consider our main model in which the seller chooses the reserve price r as well as an information structure (S, π). We begin by making a crucial observation that for revenue maximization, it suffices to consider simple binary-cutoff signals. We then characterize how the optimal reserve price depends on search costs c and also compare the seller's revenue to those of the benchmark cases studied in Section 3.

4.1 Optimal Information Structure

Recall that the seller in our model is endowed with full flexibility in advertising content, that is, she can choose any set S and a function $\pi : \mathcal{V} \to S$. This makes it practically impossible to solve for the optimal signal by directly computing her expected revenue for each signal structure: one can calculate the seller's expected revenue given bidders' participation behavior, but it seems impossible to obtain general characterization for bidders' *equilibrium* entry strategies for all possible signal structures.[13]

The following result shows that for the seller's revenue maximization, it suffices to consider a particularly simple class of signal structures.

[11] A similar result on surplus extraction is found in Crémer, Spiegel and Zheng (2009).

[12] With full information, certain bidder types necessarily obtain information rents, and thus even this constrained full-surplus extraction is impossible.

[13] For example, consider the following signal structure: $S = \{0, 1\}$ and for some $0 < v_1 < v_2 < \overline{v}$,

$$\pi(v) = \begin{cases} 0, & \text{if } v \in [0, v_1) \cup [v_2, \overline{v}], \\ 1, & \text{if } v \in [v_1, v_2). \end{cases}$$

Despite the simplicity of this signal structure, depending on the distribution function F and the values of v_1, v_2, and c, each of the following 4 (symmetric) cases can arise: each bidder (i) enters only when $s = 0$, (ii) enters only when $s = 1$, (iii) enters regardless of s, and (iv) never enters.

Theorem 1 *For a given reserve price r, the seller cannot do better than informing each bidder whether or not his value is above a certain threshold $\tilde{v}$. In other words, for each r, there exists a revenue-maximizing signal structure such that $S = \{0, 1\}$, $\pi(0|v) = 1$ if $v < \tilde{v}$, and $\pi(1|v) = 1$ if $v \geq \tilde{v}$.*

Proof. See Appendix. ■

This result is intuitive for multiple reasons. First, the signal structure in Theorem 1 is the one used for Proposition 4 (i.e., the seller's full surplus extraction with entry fees). Second, Anderson and Renault (2006) establish, and make use of, a similar result for the monopoly pricing problem (which can be interpreted as the case where there is only one bidder). Finally, conceptually, the signal affects only bidders' participation (not bidding), which is binary, and thus there seems no need for the seller to employ a more complicated signal structure. In addition, a bidder would contribute more to the seller's revenue, the higher his type is, and thus it would be optimal for the seller to attract an upper segment of bidder types.

There are several non-trivial aspects to Theorem 1, though. Among other things, it applies only to the seller's revenue maximization problem. In other words, if the information-design objective is not to maximize the seller's revenue (but to, e.g., maximize bidder surplus), this result no longer holds and it would be necessary to consider more sophisticated information structures (see, e.g., Bergemann et al., 2015; Roesler and Szentes, 2017; Choi et al., 2019). In addition, as explained shortly, formally establishing Theorem 1 (i.e., proving the two necessary properties of the optimal signal structure) requires a careful analysis of bidders' *equilibrium* participating incentives, which is absent in the social planner's problem in Section 3.3 as well as in Anderson and Renault (2006).

We prove Theorem 1 in two steps. First, we show that any signal can be replaced by a binary signal without affecting the outcome. Second, we prove that a revenue-maximizing signal must have a cutoff structure (that each bidder enters if and only if his type exceeds a certain threshold).

Step 1: replacing a signal with a binary signal. Fix a reserve price r, and consider any arbitrary signal (S, π). In any symmetric equilibrium among the bidders,[14] each bidder's entry *outcome* can be summarized by a function $\phi : \mathcal{V} \to [0, 1]$, where $\phi(v)$ represents the unconditional probability that each bidder enters when his true value v. To be formal, let $q(s)$ be the (equilibrium) probability that each bidder enters the auction after receiving $s \in S$. Then, $\phi(v)$ is given by

$$\phi(v) \equiv \int_S q(s)\pi(s|v)ds.$$

[14]Note that the subsequent argument holds even if there are multiple (symmetric) equilibria among the bidders given r and (S, π).

Consider the following alternative signal structure: $S' = \{0,1\}$ and $\pi'(1|v) = \phi(v)$. In other words, the signal is binary: each bidder receives either 0 or 1. In addition, conditional on $v \in \mathcal{V}$, each bidder receives 1 with the same probability as the probability that he enters the auction under the original signal (S, π). If each bidder enters if and only if $s = 1$ then, by construction, this signal induces the same entry outcome (and, therefore, the same expected revenue as well) as the original signal.

It still remains to show that bidders' participation incentives are preserved. This result follows from the fact that a bidder's expected payoff from entering the auction is linear in his belief. To be specific, suppose that under the original signal, a bidder is willing to participate conditional on both s_1 and s_2. Letting $u(v)$ represent his expected payoff from the auction conditional on v, this is equivalent to

$$\int u(v)dF(v|s_1), \int u(v)dF(v|s_2) \geq c.$$

Now suppose that the seller combines these two signal realizations into s, that is, the bidder cannot distinguish between s_1 and s_2. Then, after receiving s, the bidder's expected payoff is equal to

$$\int u(v)d\left(wF(v|s_1) + (1-w)F(v|s_2)\right)) \geq w\int u(v)dF(v|s_1) + (1-w)\int u(v)dF(v|s_2) \geq c$$

for some $w \in [0,1]$. In other words, the bidder is still willing to participate in the auction. The same argument applies to the opposite case: if a bidder does not enter conditional on s_1 and s_2, then he is still not willing to enter even if the two realizations are pooled. It follows that under the new signal structure (S', π'), it is an equilibrium that each bidder enters the auction if and only if $s = 1$.

Step 2: optimality of the cutoff structure. Now consider any binary signal (S, π) in which $S = \{0,1\}$ and each bidder enters the auction with a positive probability if and only if $s = 1$. We show that if this signal does not take a cutoff structure (i.e., there exists $\tilde{v}$ such that $\pi(1|v) = 1$ if $v \geq \tilde{v}$ and $\pi(0|v) = 1$ if $v < \tilde{v}$), then there necessarily exists an alternative signal that yields a higher expected revenue to the seller.

The key to this step is showing that after a change in the signal structure, bidders continue to participate in the auction in equilibrium. In other words, we show that after a change in the signal, bidders receive at least as much interim utility after a positive signal as before and therefore continue to participate at the same rate. In principle, this may or may not be the case for an arbitrary change in the signal structure. For example, consider a change that increases the probability of a bidder with value v receiving a signal of 1. There are two effects on the interim utility. On the one hand, conditional on getting $s = 1$, a bidder's expected value is larger. On the other hand, however, any bidder with a realized value less than v now has a smaller chance of winning because higher-value bidders are more likely to attend the auction.

To see more clearly how the signal structure affects bidders' participation, we define the interim utility. Let $H_S(\pi, v)$ be the probability that all other bidders either do not participate (receive $s = 0$), or do participate but have values less than v.

$$\begin{aligned} H_S(\pi, v) &= \left(\int_{\underline{v}}^{\overline{v}} (1 - \phi(x))dF(x) + \int_{\underline{v}}^{v} \phi(x)dF(x) \right)^{N-1} \\ &= \left(F(v) + \int_{v}^{\overline{v}} (1 - \phi(x))dF(x) \right)^{N-1}. \end{aligned}$$

Additionally, define $F(v|s = 1)$ as the distribution of values conditional on receiving a positive signal, or

$$F(v|s = 1) = \frac{\int_{\underline{v}}^{v} \phi(x)dF(x)}{\int_{\underline{v}}^{\overline{v}} \phi(x)dF(x)}.$$

Then, conditional on the signal structure and the reserve price, interim utility is

$$u_\pi(r) = \int_r^{\overline{v}} \left(vH_S(\pi, v) - rH_S(\pi, r) - \int_r^{v} xdH_S(\pi, x) \right) dF(v|s = 1) = \int_r^{\overline{v}} H_S(\pi, v)(1-F(v|s = 1))dv.$$

Again considering the example above, if the signal structure changes such that a value $x \in (r, \overline{v})$ receives $s = 1$ with a higher probability, then $H_S(\pi, v)$ decreases for all values less than x, while $1 - F(v|s = 1)$ increases.

We show that adjusting the probabilities of different bidder types receiving positive signals in a particular way yields at least as much utility to the interim bidder and strictly higher revenue to the seller. As for the seller, this adjustment creates a distribution of attending values which first-order stochastically dominates the original distribution of attendees. In other words, bidders with higher values are now more likely to attend, which strictly raises the expected revenue of the seller.

4.2 Optimal Advertising and Auction

We now turn to the seller's optimal choice of $(\tilde{v}, r)$ when optimally using a cutoff binary signal. Intuitively, the seller must balance her use of information ($\tilde{v}$) and guaranteed price (r). Both more information (higher $\tilde{v}$) and a lower reserve price make attendance more attractive, but as we will see, the optimal mix of information and price depends critically on the cost of attendance. Define $H_B(v) = F(v)^{N-1}$ as the probability that a bidder with value $v \geq \tilde{v}$ has the highest bid in attendance.

Proposition 5 *There exist $\overline{c} \in (0, \overline{v})$ and $\underline{c} \in (0, \overline{c})$ such that* *i) if $c \geq \overline{c}$, then the seller extracts full surplus by setting $\tilde{v} = v^*(c)$, and*

$$r = \frac{1}{H_B(v^*(c))(1 - F(v^*(c)))} \int_{v^*(c)}^{\overline{v}} H_B(v)(1 - F(v))dv,$$

ii) if $c \in (\underline{c}, \overline{c})$, then the seller chooses $\tilde{v}$ and r such that interim utility $u_B(\tilde{v}, r, 1) = c$ and $r = \tilde{v}$, and

iii) if $c \leq \underline{c}$, then the seller optimally sets $\tilde{v} = r = r_0$.

Proof. See Appendix. ■

First consider the case of high costs. Proposition 5 shows that the seller can extract full surplus if and only if c exceeds a certain threshold. To gain intuition for how the seller is able to capture the full surplus, consider the extreme case, as $c \to \overline{v}$. To incentivize attendance with such high costs, the seller must choose $(\tilde{v}, r)$ such that the interim utility conditional on a positive signal is sufficiently high. To that end, the seller decreases the reserve price as c increases. She also sets a higher and higher information cutoff $\tilde{v}$. This has a direct and an indirect effect on interim utility. Clearly, a higher $\tilde{v}$ guarantees a higher expected value to the bidder. In addition, a higher $\tilde{v}$ decreases the expected number of participating bidders, making it more likely that any bidder above the threshold is the only attendee and therefore pays r. Then, as c becomes large, $\tilde{v}$ approaches $\overline{v}$ and r approaches 0 to guarantee attendance.

The seller can only incentivize participation in this way, extracting all of the surplus, when costs are sufficiently high. As cost decreases from $\overline{v}$, $\tilde{v}$ decreases and r increases. Eventually, for low enough costs, the constraint that $\tilde{v} \geq r$ binds. This cost is what we define (in the Appendix) as $\overline{c}$.

Next consider the case of relatively low participation costs. Proposition 5 shows that the standard reserve price is optimal for costs under a threshold $\underline{c}$. To gain intuition, again consider the extreme case, this time as $c \to 0$. Then regardless of what advertisement the seller sends, all bidders attend the auction and see their values. Clearly in this case, the standard reserve price of r_0 is optimal. For costs close enough to 0, this continues to be true, i.e. when attendance is cheap enough, the seller is not concerned about attendance. Put another way, from the seller's perspective, it suffices to attract bidders with values above r, because those with values below r do not contribute to her revenue. Given this, it is optimal for the seller to set $r = r_0$ whenever it is possible to attract all bidders with values above r_0. As costs increase, this strategy becomes infeasible when the expected benefit per bidder with a reservation price of r_0 falls below c. The cutoff $\underline{c}$ is defined (in the Appendix) as the cost at which a bidder receiving a positive signal is just indifferent about attending when $\tilde{v} = r = r_0$.

Finally, suppose that $c \in (\underline{c}, \overline{c})$. The seller can neither extract full surplus nor obtain the same expected payoff as in the unconstrained case. The final part of Proposition 5 shows that the optimal combination of advertising and reserve price leaves no surplus to buyers and invites only those bidders whose values are at least r. The consumer is just indifferent about attendance for all $c \in [\underline{c}, \overline{c}]$. When cost increases past $\underline{c}$, the seller can no longer attract bidders with the strategy $\tilde{v} = r = r_0$. She needs to compensate bidders for higher costs with extra interim utility. She does not give up any more surplus than is necessary, however, to guarantee attendance, setting $u_B(\tilde{v}, r, 1) = c$.

To finish characterizing the seller's optimal choice when costs are intermediate, note that the optimality of setting the reserve price equal to the advertising cutoff when $c \in (\underline{c}, \overline{c})$ allows us to say the following.

Corollary 1 *If $c \in (\underline{c}, \overline{c})$, then $r = \tilde{v}$ is monotone in c.*

Proof. See Appendix. ■

Consider what would happen if $r = \tilde{v}$ was not monotone in c. Then, there would exist two costs, c_1 and $c_2 > c_1$ such that the same $\tilde{v} = r$ convinces bidders to attend with either cost. By proposition 5 part iii), at c_2, $u_B(\tilde{v}, r, 1) = c_2 > c_1$. However, this implies that the seller is allowing bidders strictly positive utility at c_1, contrary to proposition 5 part iii), and can do better by choosing a different level of advertising.

To guarantee participation when $c \in (\underline{c}, \overline{c})$, the seller must adjust $\tilde{v} = r$ to maintain $u_B(\tilde{v}, r, 1) = c$ as cost rises. Changing the advertising cutoff $\tilde{v} = r$ has both positive and negative effects on interim utility. On the one hand, increasing $\tilde{v} = r$ indicates more precise advertising, which positively influences expected utility conditional on a good signal. On the other hand, a higher reservation price is clearly bad for bidders. Given that $\tilde{v} = r$ is monotone over $(\underline{c}, \overline{c})$, which of these effects dominates determines whether advertising is increasing or decreasing over this range.

To determine whether the advertising or reserve price effect dominates, we must first determine how the optimal information cutoff at $\underline{c}$, r_0, compares to the efficient level, $v^*(\underline{c})$. If r_0 is larger, then $\tilde{v} = r$ must decrease over $[\underline{c}, \overline{c}]$ in order to coincide with the efficient level by $\overline{c}$. If r_0 is smaller, then the reverse must be true. To assist in this comparison, it is useful to define $\hat{c}(N) = r_0 F(r_0)^{N-1}$ as the cost at which r_0 is the efficient level of advertising as a function of N. Recall that the efficient advertising cutoff $v^*(c)$ increases more sharply in c the larger N becomes. Therefore, the cost at which r_0 is efficient decreases in N. It is also useful to think of $\underline{c}$ as a function of N. It too is decreasing in the number of potential bidders because as N increases, each bidder's chance of winning and hence his expected utility decreases. Therefore, $\underline{c}$, the highest cost for which r_0 can attract bidders, falls.

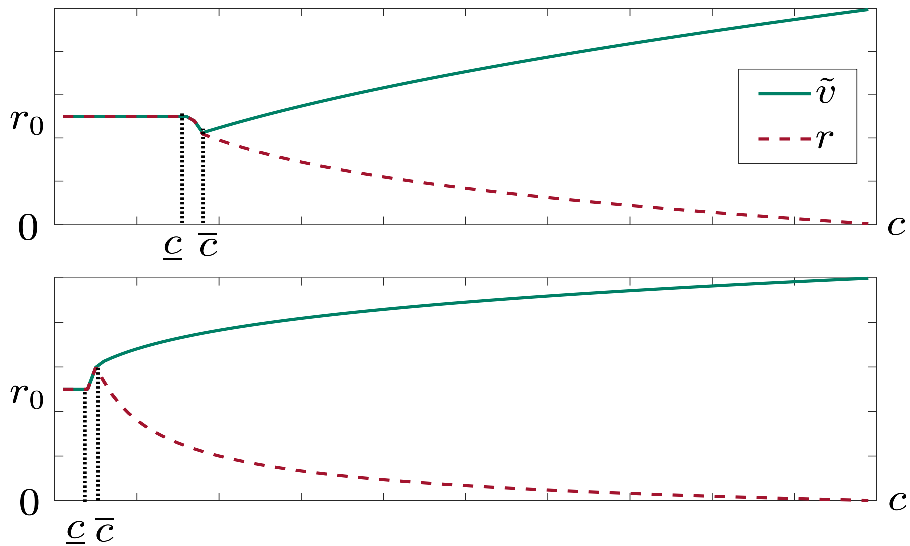

Figure 3: Optimal $\tilde{v}$ and r under the uniform distribution. Top: $N < N^*$. Bottom: $N > N^*$.

The following proposition uses $\hat{c}(N)$ and $\underline{c}(N)$ to characterize when advertising and reserve price increase and decrease over $(\underline{c}, \overline{c})$.

Proposition 6 *If*

$$r_0 \geq \frac{1}{1 - F(r_0)} \int_{r_0}^{\overline{v}} (1 - F(v))dv,$$

then there exists an N^* such that $\underline{c}(N) < \hat{c}(N) \Leftrightarrow N < N^*$, and $\underline{c}(N) > \hat{c}(N) \Leftrightarrow N > N^*$. This implies that when $c \in (\underline{c}, \overline{c})$, $\tilde{v} = r$ is decreasing for $N < N^*$, while $\tilde{v} = r$ is increasing for $N > N^*$.

Proof. See Appendix. ■

Proposition 6 says that if the number of potential bidders is relatively low, then $\tilde{v} = r$ is decreasing in c, while if it is relatively high, then $\tilde{v} = r$ is increasing in c. Intuitively, if N is small, then it is likely that any participating bidder is the only bidder and therefore pays r. Bidders therefore care more about reservation price than informative advertising and in order to raise interim utility as costs rise, the seller lowers $\tilde{v} = r$. If N is relatively large, however, then any attending bidder is likely not the only one, so that lowering the reserve price no longer increases utility. Instead, the seller raises $\tilde{v} = r$ to provide more information in order to raise u_B to match rising costs.

Mechanically, proposition 6 is true because $\hat{c}(N)$ and $\underline{c}(N)$ exhibit a single-crossing property. If the functions cross, then it is with $\underline{c}(N) < \hat{c}(N)$ to the left of N^*. In addition, both $\hat{c}(N)$ and

$\underline{c}(N)$ approach 0 as N approaches infinity. The condiiton in the proposition guarantees that $\hat{c}(N)$ is larger than $\underline{c}(N)$ for small N, and therefore that they cross at some $N^* \leq \infty$.[15] Hence, if the condition fails, then $\hat{c}(N) \leq \underline{c}(N)$ and $\tilde{v} = r$ is increasing in c for all N.[16]

To summarize, optimal advertising and reserve price are structured as follows. Consider Figure 3. If cost is sufficiently high (above $\overline{c}$), then the seller is able to capture all available surplus. She incentivizes attendance by increasing the advertising cutoff and lowering the reserve price as the cost of attendance increases. If cost is sufficiently low (below $\underline{c}$), then the seller is not concerned about incentivizing attendance and sets the standard reserve price of $\tilde{v} = r = r_0$. Finally, if cost is intermediate, then the seller lowers the reserve price for small N and raises the advertising cutoff for large N to guarantee attendance.

5 Discussion

We conclude by comparing the results from the benchmark cases in Section 3 to the seller's optimal information provision choice in Section 4, and discussing avenues for future research.

Efficiency. The characterization of optimal advertising allows us to determine how the seller's attendance cutoff, $\tilde{v}$, compares to that of the planner, v^*. Recall that v^* is equal to $\underline{v}$ at $c = 0$ (full attendance) and is increasing in c with $v^* = \overline{v}$ at $c = \overline{v}$.

Corollary 2 *If*

$$r_0 \geq \frac{1}{1 - F(r_0)} \int_{r_0}^{\overline{v}} (1 - F(v))dv,$$

then if $N < N^*$, attendance is $\begin{cases} \textit{inefficiently low} & \textit{if } c < \overline{c} \\ \textit{efficient} & \textit{if } c \geq \overline{c}, \end{cases}$

and if $N \geq N^*$, attendance is $\begin{cases} \textit{inefficiently low} & \textit{if } c \leq \hat{c} \\ \textit{inefficiently high} & \textit{if } c \in (\hat{c}, \overline{c}) \\ \textit{efficient} & \textit{if } c \geq \overline{c}. \end{cases}$

In all cases, attendance is efficient if $c \geq \overline{c}$. In addition, attendance is always inefficiently low as cost approaches zero. At such low costs, the planner would prefer for bidders with nearly all values to attend the auction. The seller, however, sets a strictly positive reserve price, restricting attendance. By proposition 6, if $N < N^*$, then $\underline{c} < \hat{c}$ so that attendance is still inefficiently low

[15]Note that this guarantees the existence of some $N^* \leq \infty$, although N^* may be equal to infinity. In that case, $\underline{c}(N) \leq \hat{c}(N)$ and $\tilde{v} = r$ is decreasing in c for all N.

[16]A large class of distributions satisfy the condition including the uniform, exponential, normal, gamma, and chi squared distributions.

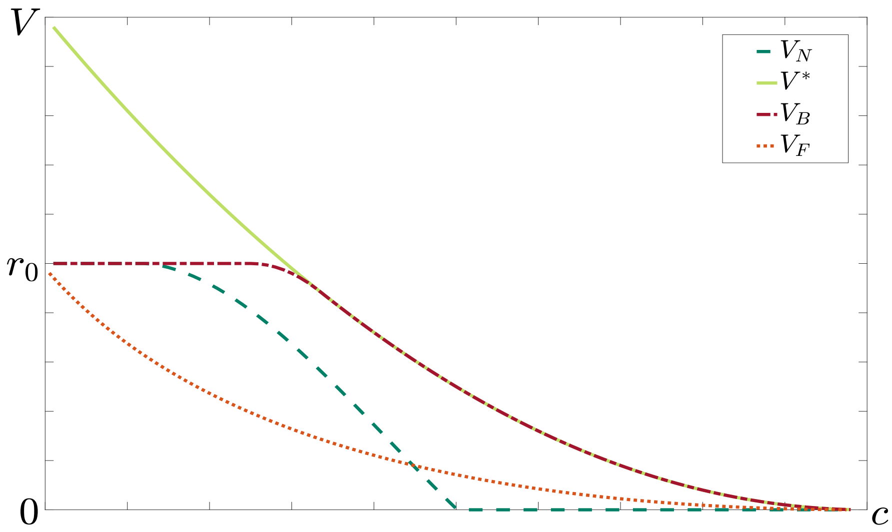

Figure 4: Revenue comparison under different information regimes.

at $\underline{c}$. As cost increases, and $\tilde{v} = r$ decreases, it remains low until $\overline{c}$. If $N > N^*$, then attendance becomes inefficiently high before $\underline{c}$ (at $\hat{c}$). It remains high as $\tilde{v} = r$ increases until $\overline{c}$.

Revenue Comparison. Consider the revenue comparison depicted in Figure 4. Let us first examine how no information relates to the optimal, binary information structure. Recall that when the bidders are given no information prior to their attendance decisions, the seller sets a reserve price of r_0 for all $c \leq \underline{c}_N$. Therefore, for those costs, attendance does not coincide; all bidders attend when there is no information while only those with values above r_0 attend if information is binary. However, because the seller does not benefit from bidders with values below r_0, revenues coincide for low costs. For higher costs, bidders eventually attend with probability 0 with no information, but the optimal information structure allows for continued attendance and therefore revenue, even when attendance costs are large through the provision of more information. Next, we see that the revenue possible from informing bidders fully of their values is strictly less that of binary information for all c. This is because full information leads to lower attendance. In other words, the attendance cutoff with full information is higher than $\tilde{v}$. Holding back some information allows the seller to obtain higher attendance and therefore higher revenue. Finally, it is clear from our analysis that the seller achieves the maximum revenue possible for $c \geq \overline{c}$. We are able to further note, however, that of the information structures considered, only the binary one allows the seller to capture full surplus for some range of costs.

Appendix: Omitted Proofs

Proof of Proposition 1 From Myerson, we know that r_0 is optimal if it produces full participation, or until $u_N(r_0, 1) = \underline{c}_N$. For costs greater than this, $u_N(r, q) = c$, as the seller does not give bidders any more surplus than necessary for participation. Consider the optimal choice of r to maximize π, conditional on q being determined by $u_N(r, c) = c$. We can use this constraint to rewrite profit as

$$V_N(c, r) = Nq\left(-c + \int_r^{\overline{v}} vH_q(v)dF(v)\right).$$

Then,

$$\frac{dV_N}{dr} = N\left(\frac{dq}{dr}\left(-c + \int_r^{\overline{v}} vH_q(v)dF(v)\right) - qrH_q(r)f(r)\right) < 0,$$

because $dq/dr < 0$ by Lemma 1, and nonnegative profit implies that $-c + \int_r^{\overline{v}} vH_q(v)dF(v) \geq 0$. Therefore, to maximize her payoff, the seller maximizing participation (keeping $q = 1$) by lowering r. This is no longer possible when the seller cannot further lower r, or for costs higher than $u_N(0, 1) = \overline{c}_N$.

Proof of Proposition 2 The seller maximizes $V_F(c, r)$ in equation (1) subject to the constraint that

$$u_F(v(c, r), r, v(c, r)) = H_{v(c,r)}(v(c, r))v(c, r) - rH_{v(c,r)}(v(c, r)) = c.$$

From the constraint, we get

$$\frac{dv(c, r)}{dr} = \frac{H_{v(c,r)}(v(c, r))}{h_{v(c,r)}(v(c, r))(v(c, r) - r) + H_{v(c,r)}(v(c, r))} \geq 0.$$

Using this, the first-order condition to the seller's revenue-maximization problem is given by

$$\begin{aligned}\frac{dV_F}{dr} =& N\left(H_{v(c,r)}(v(c, r))(1 - F(v(c, r)))\right) \\ &- N\left(\frac{dv(c, r)}{dr}(rH_{v(c,r)}(v(c, r))f(v(c, r)) + h_{v(c,r)}(v(c, r))(1 - F(v(c, r)))(v(c, r) - r))\right) = 0,\end{aligned}$$

which simplifies to

$$r_F^*(c) = \frac{1 - F(v(c, r_F^*(c)))}{f(v(c, r_F^*(c)))}.$$

The desired result that $r_F^*(c)$ is decreasing in c then follows from the fact that $v(c, r)$ is continuously increasing in c and $(1 - F)/f$ is continuously decreasing.

Proof of Proposition 5 First, we define $\overline{c}$ and $\underline{c}$.

$$\overline{c} = \frac{1}{1 - F(\phi(\overline{c}))} \int_{\phi(\overline{c})}^{\overline{v}} H_B(v)(1 - F(v))dv,$$
$$\underline{c} = \frac{1}{1 - F(r_0)} \int_{r_0}^{\overline{v}} (1 - F(v))H_B(v)dv.$$

Part i). To extract full surplus, it is necessary that $\tilde{v} = v^*(c)$, $u_B(\tilde{v}, r, 1) = c$ and $r \leq \overline{v}$. They are also sufficient, because they imply

$$u_B(\tilde{v}, r, 1) = H_B(v^*(c))v^*(c) - H_B(v^*(c))r + \frac{1}{1 - F(v^*(c))} \int_{v^*(c)}^{\overline{v}} H_B(v)(1 - F(v))dv = c,$$

which leads to

$$\begin{aligned} V_B(\tilde{v}, r) &= N\left((1 - F(v^*(c)))H_B(v^*(c))r + \int_{v^*(c)}^{\overline{v}} (1 - F(v))vh_B(v)dv\right) \\ &= N\left(\int_{v^*(c)}^{\overline{v}} H_B(v)(1 - F(v))dv + \int_{v^*(c)}^{\overline{v}} (1 - F(v))vh_B(v)dv\right) \\ &= N\left(-(1 - F(v^*(c)))c + \int_{v^*(c)}^{\overline{v}} dH_B(v)dF(v)\right), \end{aligned}$$

which is the highest available surplus. The only potentially binding condition is $r \leq \overline{v}$. Given that r must take the form given in the statement, $c \geq \overline{c}$ is necessary and sufficient for $r \leq \overline{v}$.

Part ii). We first show that the interim bidder must be indifferent about attendance. Suppose that $u_B(\tilde{v}, r, 1) > c$. If $\tilde{v} > r$, then it increases the seller's revenue to marginally raise r, because it does not affect bidders' participation (thus social surplus) but lowers bidder surplus (i.e., $\partial u_B / \partial r < 0$).

Now consider the case where $\tilde{v} = r$. The seller's revenue in this case is identical to her revenue with reserve price r in the unconstrained case (without participation costs). Therefore, unless $r = r_0$, the seller can increase her revenue by moving r closer to r_0 (given that the seller's revenue in the unconstrained case is quasi-concave in r). However, by part iii), $\tilde{v} = r = r_0$ is not implementable when $c \in (\underline{c}, \overline{c})$.

This implies that when $c \in (\underline{c}, \overline{c})$, the seller faces the following constrained optimization problem:

$$\max_{\tilde{v}, r} N\left((1 - F(\tilde{v}))H_B(\tilde{v})r + \int_{\tilde{v}}^{\overline{v}} (1 - F(v))h_B(v)vdv\right)$$

subject to $u_B = c$ and $r \leq \tilde{v}$. By the implicit function theorem, the first constraint can be replaced by

$$\frac{d\tilde{v}}{dr} = \frac{H_B(\tilde{v})}{(\tilde{v}-r)h_B(\tilde{v}) + \frac{f(\tilde{v})}{(1-F(\tilde{v}))^2}\int_{\tilde{v}}^{\overline{v}}(1-F(v))H_B(v)dv}.$$

The associated Lagrangian is

$$\mathcal{L} = N\left((1-F(\tilde{v}))rH_B(\tilde{v}) + \int_{\tilde{v}}^{\overline{v}}(1-F(v))vh_B(v)dv\right) - \mu(r-\tilde{v}).$$

Clearly, it must be that $\mu(r-\tilde{v}) = 0$. In addition, the first-order condition with respect to r is

$$\begin{aligned}
&N\left(H_B(\tilde{v})(1-F(\tilde{v})) + \frac{d\tilde{v}}{dr}\left(rh_B(\tilde{v})(1-F(\tilde{v})) - rH_B(\tilde{v})f(\tilde{v}) - (1-F(\tilde{v}))h_B(\tilde{v})\tilde{v}\right)\right) - \mu\left(1-\frac{d\tilde{v}}{dr}\right)\\
=&\frac{N}{(\tilde{v}-r)h_B(\tilde{v}) + \frac{f(\tilde{v})}{(1-F(\tilde{v}))^2}\int_{\tilde{v}}^{\overline{v}}(1-F(v))H_B(v)dv}\left(H_B(\tilde{v})(1-F(\tilde{v}))h_B(\tilde{v})(\tilde{v}-r)\right)\\
&+\frac{N}{(\tilde{v}-r)h_B(\tilde{v}) + \frac{f(\tilde{v})}{(1-F(\tilde{v}))^2}\int_{\tilde{v}}^{\overline{v}}(1-F(v))H_B(v)dv}\left(\frac{H_B(\tilde{v})f(\tilde{v})}{1-F(\tilde{v})}\int_{\tilde{v}}^{\overline{v}}(1-F(v))H_B(v)dv\right)\\
&+\frac{N}{(\tilde{v}-r)h_B(\tilde{v}) + \frac{f(\tilde{v})}{(1-F(\tilde{v}))^2}\int_{\tilde{v}}^{\overline{v}}(1-F(v))H_B(v)dv}\left(H_B(\tilde{v})h(\tilde{v})(1-F(\tilde{v}))(r-\tilde{v}) - rf(\tilde{v})H_B^2(\tilde{v})\right)\\
&-\mu\left(1-\frac{d\tilde{v}}{dr}\right)\\
=&N\frac{d\tilde{v}}{dr}f(\tilde{v})\left(\frac{1}{1-F(\tilde{v})}\int_{\tilde{v}}^{\overline{v}}(1-F(v))H_B(v)dv - rH_B(\tilde{v})\right) - \mu\left(1-\frac{d\tilde{v}}{dr}\right)\\
=&N\frac{d\tilde{v}}{dr}f(\tilde{v})(c-\tilde{v}H_B(\tilde{v})) - \mu\left(1-\frac{d\tilde{v}}{dr}\right)
\end{aligned}$$

where the first equality is obtained by substituting in from the $\frac{d\tilde{v}}{dr}$ constraint, and the third equality is obtain by substituting in from $u_B = c$ for the integral.

So, if $r < \tilde{v}$ then $\mu = 0$, and $c - \tilde{v}H_B(\tilde{v}) = 0$. This, however, is only feasible for $c \geq \overline{c}$. So it must be that $r = \tilde{v}$ and $\mu > 0$.

Part iii). The optimal reserve price being r_0 for costs below $\underline{c}$ is immediate from the fact that the seller's revenue when she can set only r cannot exceed her revenue under an optimal mechanism without attendance costs.

Finally, to show that $\underline{c} \leq \overline{c}$, suppose not, that $\overline{c} < \underline{c}$. Then for all $c \in (\overline{c}, \underline{c})$, the seller can capture full surplus by part i). However, because $r_0 < u_B(r_0, r_0, 1)$ for $c < \underline{c}$, setting a reserve price of r_0 in this range indicates that the seller is not capturing full surplus. This contradicts the optimality of $\tilde{v} = r = r_0$.

Proof of Corollary 1 We set the first-order condition from the proof of proposition 5, part ii) equal to zero to obtain

$$\mu = \frac{\frac{d\tilde{v}}{dr} N f(\tilde{v})(c - \tilde{v} H_B(\tilde{v}))}{1 - \frac{d\tilde{v}}{dr}} = \frac{H_B(\tilde{v}) N f(\tilde{v})(c - \tilde{v} H_B(\tilde{v}))}{\frac{f(\tilde{v})}{(1-F(\tilde{v}))^2} \int_{\tilde{v}}^{\overline{v}} (1 - F(v)) H_B(v) dv - H_B(\tilde{v})} = \frac{H_B(\tilde{v}) N f(\tilde{v})(c - \tilde{v} H_B(\tilde{v}))}{\frac{f(\tilde{v})c}{1-F(\tilde{v})} - H_B(\tilde{v})},$$

where the last equality again comes from substituting in from $u_B = c$ for the integral. Then because $\tilde{v} = r$,

$$\frac{du_B}{d\tilde{v}} = \frac{f(\tilde{v})}{(1 - F(\tilde{v}))^2} \int_{\tilde{v}}^{\overline{v}} (1 - F(v)) H_B(v) dv - H_B(\tilde{v}) = \frac{f(\tilde{v})c}{1 - F(\tilde{v})} - H_B(\tilde{v}), \qquad (2)$$

so that

$$\mu = \frac{H_B(\tilde{v}) N f(\tilde{v})(c - \tilde{v} H_B(\tilde{v}))}{\frac{du_B}{d\tilde{v}}}.$$

Therefore, to ensure $\mu > 0$, $\frac{du_B}{d\tilde{v}}(\tilde{v} = r) > 0 \Leftrightarrow c > H_B(\tilde{v})\tilde{v}$ and $\frac{du_B}{d\tilde{v}}(\tilde{v} = r) < 0 \Leftrightarrow c < H_B(\tilde{v})\tilde{v}$. Therefore, if $\tilde{v} = r$ was not monotone then there would have to be at least one place where $\frac{du_B}{d\tilde{v}}(\tilde{v} = r) > 0$ but $c < H_B(\tilde{v})\tilde{v}$ or visa versa.

Proof of Proposition 6 **Claim:** $\hat{c}(N)$ and $\underline{c}(N)$ cross at most once. If they do cross, then $\hat{c}(N)$ crosses from above to below $\underline{c}(N)$.
Both $\underline{c}(N)$ and $\hat{c}(N)$ are decreasing and convex in N.

$$\begin{aligned}
\hat{c} &= r_0 F(r_0)^{N-1} \Rightarrow \frac{d\hat{c}}{dN} = r_0 F(r_0)^{N-1} log(F(r_0)) < 0 \\
\frac{d^2\hat{c}}{dN^2} &= r_0 F(r_0)^{N-1} (log(F(r_0)))^2 > 0 \\
\underline{c} &= \frac{1}{1 - F(r_0)} \int_{r_0}^{\overline{v}} (1 - F(v)) H_B(v) dv \Rightarrow \frac{d\underline{c}}{dN} = \frac{1}{1 - F(r_0)} \int_{r_0}^{\overline{v}} (1 - F(v)) H_B(v) log(F(v)) dv < 0 \\
\frac{d^2\underline{c}}{dN^2} &= \frac{1}{1 - F(r_0)} \int_{r_0}^{\overline{v}} (1 - F(v)) H_B(v) (log(F(v)))^2 dv > 0.
\end{aligned}$$

To compare the slopes at any point at which they cross, note that

$$\frac{d\hat{c}}{dN}(\hat{c} = \underline{c}) = \frac{log(F(r_0))}{1 - F(r_0)} \int_{r_0}^{\overline{v}} (1 - F(v)) H_B(v) dv.$$

At any point at which the functions cross, $\hat{c}$ is more steeply sloped than $\underline{c}$ because

$$\left| \log(F(r_0)) \int_{r_0}^{\overline{v}} (1 - F(v)) H_B(v) dv \right| > \left| \int_{r_0}^{\overline{v}} (1 - F(v)) H_B(v) log(F(v)) dv \right|.$$

This is because $log(F(v)) < 0$ for all v, but is the most negative (highest absolute value) at r_0. We now need to compare $\hat{c}$ and $\underline{c}$ at extreme levels of N. Both $\hat{c}$ and $\underline{c}$ approach 0 as $N \to \infty$. For $N = 1$, $\hat{c}(1) = r_0$, and

$$\underline{c}(1) = \frac{1}{1 - F(r_0)} \int_{r_0}^{\overline{v}} (1 - F(v))dv = -r_0 + \frac{1}{1 - F(r_0)} \int_{r_0}^{\overline{v}} vdF(v),$$

which is obtained by integration by parts. Then if $\hat{c}(1) \geq \underline{c}(1)$, then $\hat{c}$ and $\underline{c}$ can cross at an N^*. If $\hat{c}(1) < \underline{c}(1)$, then the functions can never cross. To verify how this is related to the direction of $\tilde{v} = r$, we show the following.

Claim:

$$\underline{c}(N) < \hat{c}(N) \Leftrightarrow \frac{du_B}{d\tilde{v}}(r = \tilde{v}) < 0, \qquad \underline{c}(N) > \hat{c}(N) \Leftrightarrow \frac{du_B}{d\tilde{v}}(r = \tilde{v}) > 0.$$

Simply plugging in $U^I(r_0, r_0) = \underline{c}$ shows that this is true. Put r_0 into (2) to get

$$\begin{aligned}
\frac{du_B}{d\tilde{v}} &= \frac{f(r_0)}{(1 - F(r_0))^2} \int_{r_0}^{\overline{v}} (1 - F(v)) H_B(v) dv - H_B(r_0) \\
&= \frac{f(r_0)}{1 - F(r_0)} \underline{c} - H_B(r_0) \\
&= \frac{\underline{c}}{r_0} - H_B(r_0) \\
&= \frac{1}{r_0}(\underline{c} - \hat{c}).
\end{aligned}$$

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